Economics 325–003 **Term I 2013**

Introduction to Empirical Economics **Hiro Kasahara**

Final Exam

1. (30 points) State whether each of the following is true or false (No explanation necessary).

   (a) (5 points) The larger the sample size, the smaller the variance of the sample standard deviation.

   (b) (5 points) The central limit theorem is not applied to discrete random variables.

   (c) (5 points) If the null hypothesis is not rejected based on sample evidence, the researcher has proven beyond a doubt that the null hypothesis is true.

   (d) (5 points) If we are interested in estimating the population mean, it is possible to construct an estimator of the population mean of which variance is lower than the variance of the sample mean.

   (e) (5 points) Given a realized sample, the confidence interval contains the population parameter with probability either one or zero.

   (f) (5 points) Let $\{X_1, X_2, ..., X_n\}$ be n observations, each of which is randomly drawn from a normal distribution with mean μ and variance σ^2. Let $s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2}$ and $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. Then, the distribution of a statistic $\frac{\bar{X}-\mu}{s/\sqrt{n}}$ is always given by t-distribution with the degree of freedom $n-1$.

2. (12 points) Let X_1, X_2, and X_3 be a random sample of observations from a population with mean μ and variance σ^2. Consider the following two point estimators of μ: $\hat{\theta}_1 = a_1X_1 + a_2X_2 + a_3X_3$ and $\hat{\theta}_2 = b_1X_1 + b_2X_2 + b_3X_3$, where $a_1, a_2, a_3, b_1, b_2, b_3$ are some constant.

   (a) (6 points) What are the conditions on $a_1, a_2, a_3, b_1, b_2, b_3$ under which both $\hat{\theta}_1$ and $\hat{\theta}_2$ are unbiased estimators of μ?

   (b) (6 points) What are the conditions on $a_1, a_2, a_3, b_1, b_2, b_3$ under which $Var(\hat{\theta}_1) < Var(\hat{\theta}_2)$ holds.

3. (6 points) In a recent survey of 600 adults, 16.4% indicated that they had fallen asleep in front of the television in the past month. Develop a 95% confidence interval for the population proportion.

4. (6 points) A student records the time (in minutes) it takes to commute to school for seven days. Those results are: 21, 15, 13, 16, 10, 13, and 18. Assuming the population is normally distributed; develop a 95% confidence interval for the population mean.

5. (6 points) Multiple Choice (No explanation necessary): An exit poll research firm claims that the proportion of Democratic voters in Salt Lake City is at most 40 percent. A random sample of 175 voters was selected and found to consist of 35 percent Democrats. Take the opposite of the research firm's claim as the null hypothesis so that $H_0 : p \geq 0.4$, where p is the population proportion. What is the p-value for this test?

   A) 0.0312 B) 0.0624 C) 0.0443 D) 0.0885

6. (8 points) Let Z_1, Z_2, and Z_3 are three Bernoulli random variables with the probability of success p, where Z_1, Z_2, and Z_3 are independent, and $Z_i = 0$ with probability $1 - p$ and $Z_i = 1$ with probability p for $i = 1, 2, 3$. Define a random variable $X = Z_1 + Z_2 + Z_3$. Prove that $Var(X) = 3p(1 - p)$.

7. (26 points) Let $\{X_1, X_2, ..., X_n\}$ be $n = 25$ observations, each of which is randomly drawn from normal distribution with mean μ and variance σ^2. The value of μ is not known while σ^2 is known and equal to 100. We are interested in testing the null hypothesis $H_0 : \mu = 5$ against the alternative hypothesis $H_1 : \mu < 5$. Consider hypothesis testing based on the following two different test statistics: (i) $\tilde{X} = 0.76X_1 + \sum_{i=2}^{25} 0.01X_i = 0.76X_1 + 0.01X_2 + 0.01X_3 + ... + 0.01X_{25}$ and (ii) $\hat{X} = (1/4)(X_1 + X_2 + X_3 + X_4)$. Suppose that the realized values of $\tilde{X}$ and $\hat{X}$ are given by $\tilde{X} = -4$ and $\hat{X} = -2$, respectively.

   (a) (6 points) Test the null hypothesis $H_0 : \mu = 5$ against $H_1 : \mu < 5$ *using the test statistic* $\tilde{X}$ at the significance level $\alpha = 0.1$.

   (b) (6 points) Compute the p-value for testing the null hypothesis $H_0 : \mu = 5$ against $H_1 : \mu < 5$ *using the test statistic* $\hat{X}$.

   (c) (4 points) True or False (No explanation necessary): *When we use the test statistic* $\hat{X}$, the null hypothesis $H_0 : \mu = 5$ against $H_1 : \mu < 5$ is rejected at the significance level $\alpha = 0.1$.

   (d) (6 points) Compute the power of test using *using the test statistic* $\tilde{X}$ when the true value of μ is equal to -2.

   (e) (4 points) True or False (No explanation necessary): comparing the power of test using $\tilde{X}$ and the power of test using $\hat{X}$ when the true value of μ is equal to -2, the power of test using $\hat{X}$ is higher than the power of test using $\tilde{X}$ and, therefore, the test using $\hat{X}$ is recommended in this case.

8. (6 points) Prove the following result: $Cov(a_1 + b_1X, a_2 + b_2Y) = b_1b_2E[XY] - b_1b_2E[X]E[Y]$, where a_1, a_2, b_1, and b_2 are some constant.