Economics 325–003 **Term I 2014**

Introduction to Empirical Economics **Hiro Kasahara**

Final Exam

1. (32 points) State whether each of the following is true or false. [No Explanation Necessary]

   (a) The larger the sample size, the smaller the variance of the sample standard deviation.

   (b) The central limit theorem is not applied to discrete random variables.

   (c) The sample standard deviation is an example of an estimator.

   (d) The p-value of two-sided test is not equal to the p-value of one-sided test.

   (e) If the p-value of a two sided test for the mean of a population is .005, then the null hypothesis will be rejected at a level of significance of .01.

   (f) If the null hypothesis is not rejected based on sample evidence, the researcher has proven beyond a doubt that the null hypothesis is true.

   (g) If a null hypothesis is rejected against an alternative at the 5% level, then using the same data, it must be rejected against that alternative at the 10% level.

   (h) As the sample size increases to infinity, the variance of the sample mean approaches zero.

2. Let X_1, X_2, X_3, and X_4 be a random sample of observations from a population with mean μ and variance σ^2. Consider the following two point estimators of μ: $\hat{\theta}_1 = 0.25X_1 + 0.25X_2 + 0.40X_3 + 0.10X_4$ and $\hat{\theta}_2 = 0.20X_1 + 0.30X_2 + 0.30X_3 + 0.20X_4$.

   (a) (4 points) Which of the following is true? [No Explanation Necessary]
   (1) $\hat{\theta}_1$ is biased, but $\hat{\theta}_2$ is unbiased estimator of μ. (2) $\hat{\theta}_1$ is unbiased, but $\hat{\theta}_2$ is biased estimator of μ. (3) Both $\hat{\theta}_1$ and $\hat{\theta}_2$ are unbiased estimators of μ. (4) Both $\hat{\theta}_1$ and $\hat{\theta}_2$ are biased estimators of μ.

(b) (4 points) Which of the following is true? [No Explanation Necessary]

(1) $Var(\hat{\theta}_1) = Var(\hat{\theta}_2)$. (2) $Var(\hat{\theta}_1) > Var(\hat{\theta}_2)$. (3) $Var(\hat{\theta}_1) < Var(\hat{\theta}_2)$. (4) Cannot tell the relationship between $Var(\hat{\theta}_1)$ and $Var(\hat{\theta}_2)$.

3. (6 points) Assume that the yield per acre for a particular variety of soybeans is $N(\mu, \sigma^2)$. For a random sample of $n = 4$ plots, the yields per acre were 37.4, 48.8, 46.9, and 55.0. Find a 90 percent confidence interval for μ.

4. If a newborn baby has a birth weigh that is less than 2500 grams, we say that the baby has a low birth weight. The proportion of babies with a low birth weight is an indicator of lack of nutrition for the mothers. In Canada, approximately 7 percent of babies have a low birth weight. Let p equal the proportion of babies born in the Sudan who weight less than 2500 grams. In a random sample of $n = 209$ babies in the Sudan, there are 23 babies whose weight is less than 2500 grams.

   (a) (6 points) Test the null hypothesis $H_0 : p \leq 0.07$ against the alternative hypothesis $H_1 : p > 0.07$ at a significance level of $\alpha = 0.05$.

   (b) (6 points) Find the p-value for this test.

5. Let X, Y, and Z are three Bernoulli random variables that are independent to each other with the probability of success p_x, p_y, and p_z, respectively. We are interested in testing the null hypothesis $H_0 : p_x + p_y \geq p_z$ against the alternative hypothesis $H_1 : p_x + p_y < p_z$. Suppose that we have a random sample of X, Y, and Z with sample size $n = 100$: $\{X_1, ..., X_n\}$, $\{Y_1, ..., Y_n\}$, and $\{Z_1, ..., Z_n\}$. The sample averages (i.e., sample proportions) of X, Y, and Z are given by $\hat{p}_x = (1/n)\sum_{i=1}^n X_i = 0.1$, $\hat{p}_y = (1/n)\sum_{i=1}^n Y_i = 0.3$, and $\hat{p}_z = (1/n)\sum_{i=1}^n Z_i = 0.5$. Denote the true value of p_x, p_y, and p_z by p_x^0, p_y^0, and p_z^0,

   (a) (6 points) What is the estimated variance of $\hat{p}_x + \hat{p}_y - \hat{p}_z$?

   (b) (6 points) Test the null hypothesis $H_0 : p_x + p_y \geq p_z$ against the alternative hypothesis $H_1 : p_x + p_y < p_z$ at the significance level $\alpha = 0.1$. [Hint: you can use sample proportions $\hat{p}_x$, $\hat{p}_y$, and $\hat{p}_z$ in place of p_x^0, p_y^0, and p_y^0 to compute the standard deviation for $\hat{p}_x + \hat{p}_y - \hat{p}_z$ when you construct a test statistic.]

6. Let $\{X_1, X_2, ..., X_n\}$ be $n = 9$ observations, each of which is randomly drawn from normal distribution with mean μ and variance σ^2. The value of μ is not known while σ^2 is known and equal to 25. We are interested in testing the null hypothesis $H_0 : \mu = 10$ against the alternative hypothesis $H_1 : \mu < 10$. Consider hypothesis testing based on the following two different test statistics: (i) sample mean $\bar{X} = (1/n) \sum_{i=1}^{n} X_i$ and (ii) weighted average $\hat{X} = 0.6X_1 + 0.05 \times \sum_{i=2}^{9} X_i$. Suppose that the realized values of $\bar{X}$ and $\hat{X}$ are given by $\bar{X} = 7.5$ and $\hat{X} = 6.5$, respectively.

   (a) (6 points) Test the null hypothesis $H_0 : \mu = 10$ against $H_1 : \mu < 10$ *using the test statistic* $\hat{X}$ at the significance level $\alpha = 0.1$. Similarly, test the null hypothesis *using the test statistic* $\bar{X}$.

   (b) (6 points) Compute (i) the power of test using *using the test statistic* $\bar{X}$ when the true value of μ is equal to 8 and (ii) the power of test *using the test statistic* $\hat{X}$ when the true value of μ is equal to 8. Based on the power comparison, which test statistics, $\bar{X}$ or $\hat{X}$, do you recommend using for hypothesis testing?

7. (6 points) Suppose X_i for $i = 1, ..., n$ is randomly drawn from normal distribution with mean μ and variance σ^2. Let $\bar{X} = (1/n) \sum_{i=1}^{n} X_i$ be the sample average. Prove that $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$.

8. (6 points) Let X be a random variable with $E[X] = \mu$ and $\text{Var}(X) = \sigma^2$. Define $Z = \frac{X-\mu}{\sigma}$. Prove that $E[Z] = 0$ and $\text{Var}(Z) = 1$.

9. (6 points) The chi-square distribution with the r degree of freedom, denoted by $\chi^2(r)$, is characterized by the sum of r independent standard normally distributed random variables Z_1^2, Z_2^2, ..., Z_r^2, where $Z_i \sim N(0, 1)$ and Z_i and Z_j are independent if $i \neq j$ for $i, j = 1, ..., r$. Namely, $W = Z_1^2 + Z_2^2 + ... + Z_r^2$ has a distribution that is $\chi^2(r)$. Prove that, if W has a $\chi^2(r)$ distribution, then $E[W] = r$.