Econ 325: Introduction to Empirical Economics

Chapter 9

Hypothesis Testing:
Single Population

What is a Hypothesis?

- A hypothesis is a claim (assumption) about a population parameter:

  - population mean

Example: The mean monthly cell phone bill of this city is $\mu = \$42$

  - population proportion

Example: The proportion of voters who support Trump is $p \geq .50$

The Null Hypothesis, H_0

- The assumption to be tested in population parameter

Example: the **population fraction** of voters who support Donald Trump is greater than or equal to 0.5

$H_0: p \geq 0.5$

$H_0: \hat{p} \geq 0.5$

The Null Hypothesis, H_0

(continued)

- Begin with the assumption that the null hypothesis is true
  - Similar to the notion of innocent until proven guilty

- Refers to the status quo
- Always contains "=" , "≤" or "≥" sign

The Alternative Hypothesis, H_1

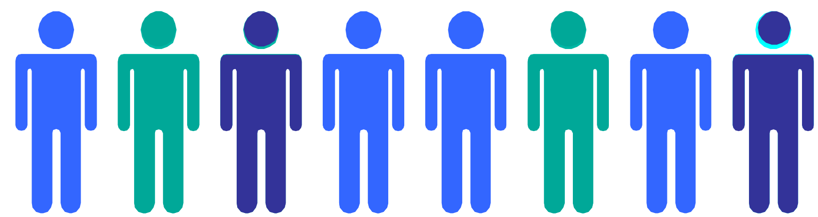
- Is the opposite of the null hypothesis
  - e.g., The population fraction of voters who support Trump is smaller than 0.5 ($H_1: p < 0.5$)
- Challenges the status quo
- Never contains the "=" , "≤" or "≥" sign
- Is generally the hypothesis that the researcher is trying to support

Example

- US presidential election: Trump vs. Clinton
- p = population fraction of Clinton supporters
- You would like to provide evidence that Clinton will win.
- $H_0: p \leq 0.5$
- Rejecting $H_0: p \leq 0.5$ gives evidence for $p > 0.5$

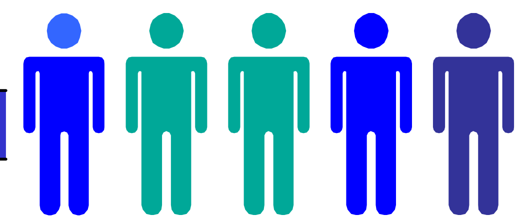
Hypothesis Testing Process

Claim: **the population mean age is 50.**
(Null Hypothesis:
H_0: $\mu = 50$)



Population

Now select a random sample



Sample

Suppose the sample mean age is 20: $\bar{X} = 20$

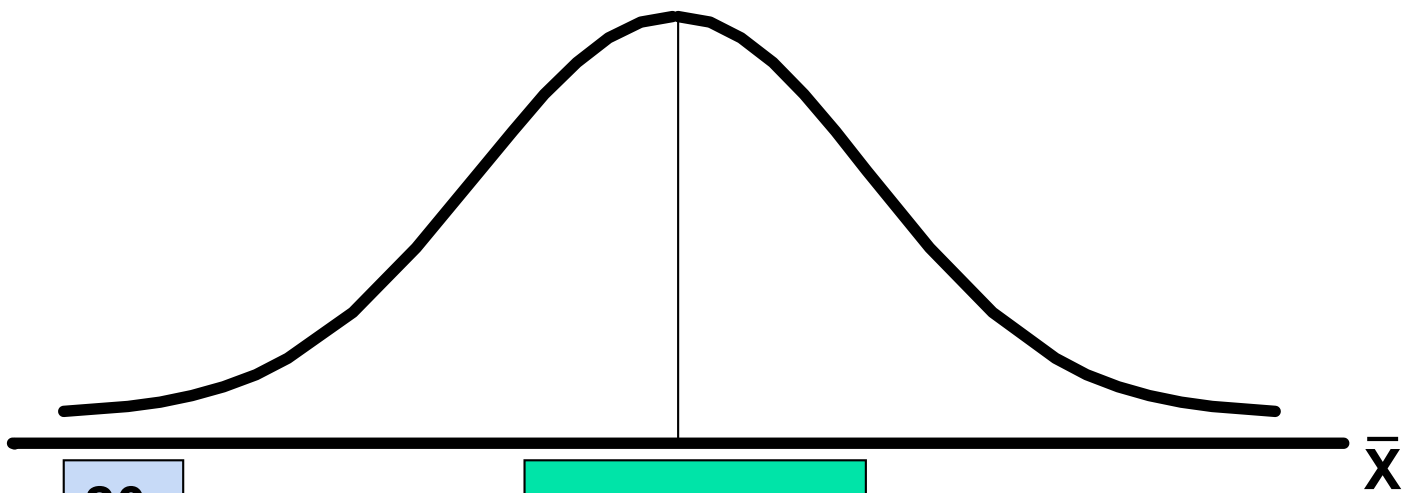
Is $\bar{X} = 20$ likely if $\mu = 50$?

If not likely,

REJECT
Null Hypothesis

Reason for Rejecting H_0

Sampling Distribution of $\bar{X}$



20

$\mu = 50$
If H_0 is true

If it is unlikely that we would get a sample mean of this value ...

... if in fact this were the population mean…

... then we reject the null hypothesis that $\mu = 50$.

Example

- On Oct 24 of 2016, the survey was conducted in Florida after the final presidential debate.
- p = population fraction of Clinton supporters
- Among $n = 1166$ likely registered voters, there are 602 Clinton voters and 564 Trump voters so that $\hat{p} = 0.516$
- Does this provide strong evidence against the null hypothesis $H_0: p \leq 0.5$?

Clicker Question 9.1

- Among $n = 1166$ voters, there are 602 Clinton voters and 564 Trump voters so that $\hat{p} = 0.516.$ Does this provide strong evidence against $H_0: p \le 0.5$?

A). Yes, it provides strong evidence against $H_0: p \le 0.5$.

B). No, it does not provide strong evidence against $H_0: p \le 0.5.$

Testing the Null Hypothesis

- Step 1: Find the distribution of $\hat{p}$ if the null hypothesis of $p = 0.5$ is true.
- Step 2: Define the values of $\hat{p}$ that are unlikely to happen if $H_0: p \leq 0.5$ is true.
- Step 3: Look at the realized value of $\hat{p}$ and check if the realized value of $\hat{p}$ is likely or not if $H_0: p \leq 0.5$ is true.
- Step 4: If you find that the realized value of $\hat{p}$ is unlikely when $H_0: p = 0.5$ is true, reject $H_0: p \leq 0.5$.

Step 1

The distribution of $\hat{p}$ when $p = 0.5$ and $n = 1166$:

$$\frac{\hat{p} - 0.5}{\sqrt{0.5(1-0.5)/1166}} = \frac{\hat{p} - 0.5}{0.015} \sim N(0,1)$$

so that

$$\hat{p} \sim N(0.5, (0.015)^2)$$

Step 2

- We define the value of $\hat{p}$ that are not likely to happen when $H_0: p = 0.5$ is true by finding a constant C (critical value) such that

$$\mathrm{P}(\hat{p} > C) = 0.05 \quad \text{(significance level)}$$

Because $P\left(\frac{\hat{p}-0.5}{0.015} > 1.64\right) = 0.05,$

$$\frac{C - 0.5}{0.015} = 1.64$$

$$\rightarrow \quad C = 0.5 + 1.64 \times 0.015 = 0.525$$

Steps 3 and 4

Therefore, the range of values called

$$\text{Rejection region} = [0.525, \infty)$$

happens with probability less than or equal to 0.05 when $H_0: p = 0.5$ is true.

$\hat{p} = 0.516$ does not fall in $[0.525, \infty)$. Therefore, there is not strong enough evidence against $H_0: p = 0.5$.

Therefore, we do not reject $H_0: p = 0.5$.

Level of Significance, α

- **Defines how strong the evidence against the null hypothesis should be for a researcher to reject the null hypothesis.**

- Is designated by α , (level of significance)
  - Typical values are .01, .05, or .10

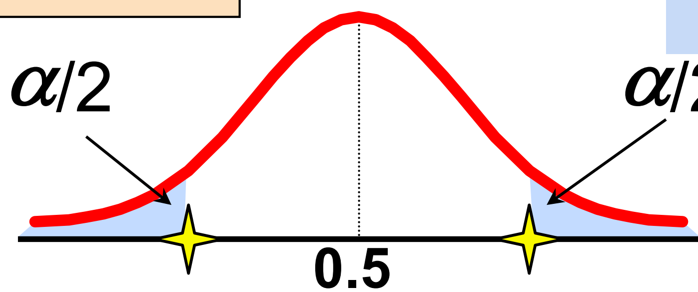
- Provides the critical value(s) of the test

Level of Significance and the Rejection Region

Level of significance = α

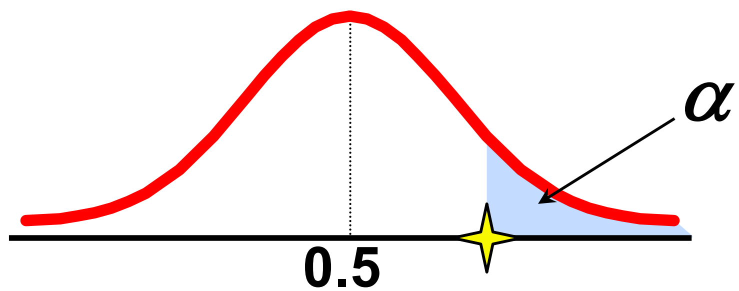
H_0: $p = 0.5$
H_1: $p \neq 0.5$

Two-tail test



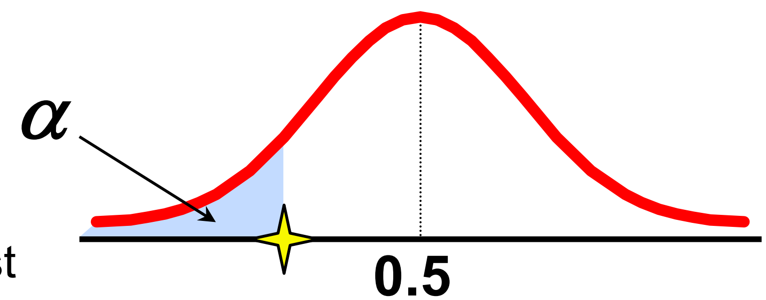
H_0: $p \leq 0.5$
H_1: $p > 0.5$

Upper-tail test



H_0: $p \geq 0.5$
H_1: $p < 0.5$

Lower-tail test



✦ **Represents critical value**

Rejection region is shaded

Example

In this example,

- Significance level is $\alpha = 0.05$
- Rejection region is $[0.525, \infty)$
- Critical value is 0.525
- The null hypothesis was not rejected because $\hat{p} = 0.516$ was outside of rejection region $[0.525, \infty)$.

Worksheet Question

- On Oct 24 of 2016, the survey was conducted in Florida after the final presidential debate.
- p = population fraction of Clinton supporters
- Among $n = 1166$ likely registered voters, there are 602 Clinton voters and 564 Trump voters so that $\hat{p} = 0.516$
- **Test $H_0: p \leq 0.5$ at the significance level $\alpha = 0.10$**

Testing the Null Hypothesis

- Step 1: Find the distribution of $\hat{p}$ if the null hypothesis of $p = 0.5$ is true.
- Step 2: Define the values of $\hat{p}$ that are unlikely to happen if $H_0: p \leq 0.5$ is true.
- Step 3: Look at the realized value of $\hat{p}$ and check if the realized value of $\hat{p}$ is likely or not if $H_0: p \leq 0.5$ is true.
- Step 4: If you find that the realized value of $\hat{p}$ is unlikely when $H_0: p = 0.5$ is true, reject $H_0: p \leq 0.5$.

Example

- If we use the significance level of $\alpha = 0.10$ instead, we do not reject the null hypothesis of $H_0: p \leq 0.5$ in the previous example:

Rejection region: $[0.5192, \infty)$

$\hat{p} = 0.516$ not in $[0.5192, \infty)$ $\rightarrow$ Do not reject $H_0: p \leq 0.5$

Clicker Question 9.2

In this example, we did not reject $H_0: p \leq 0.5$, where p = population fraction of Clinton voters. This means:

A). Clinton will not win Florida for sure.

B). Trump will not win Florida for sure.

C). There is no strong evidence that Clinton will win Florida; the evidence is inconclusive.

Example

What if the significance level is $\boldsymbol{\alpha = 0.20}$?

Because $P\left(\frac{\hat{p}-0.5}{0.015} > 0.84\right) = 0.20,$

$$\frac{C - 0.5}{0.015} = 0.84$$

$$\rightarrow \quad C = 0.5 + 0.84 \times 0.015 = 0.512$$

$\hat{p} = 0.516$ is in $[0.512, \infty)$ → Reject $H_0: p \leq 0.5$

Clicker Question 9.3

In this example, we rejected $H_0: p \leq 0.5$, where p = population fraction of Clinton voters. This means:

A). Clinton will win Florida for sure.

B). Trump will win Florida for sure.

C). There is evidence that Clinton will win Florida but, in reality, Clinton could lose Florida with a small probability.

Significance level α and Decision

- Survey in Florida: $n = 1166$ and $\hat{p} = 0.516$
- Testing $H_0: p \leq 0.5$
  - $\boldsymbol{\alpha = 0.10}$: $\hat{p} = 0.516$ not in $[0.5192, \infty)$

    $\rightarrow$ Do not reject $H_0: p \leq 0.5$
  - $\boldsymbol{\alpha = 0.20}$: $\hat{p} = 0.516$ is in $[0.512, \infty)$

    $\rightarrow$ Reject $H_0: p \leq 0.5$

Errors in Making Decisions

- **Type I Error**
  - Reject a true null hypothesis
  - Considered a serious type of error

The probability of Type I Error is α

- Called level of significance of the test
- Set by researcher in advance

Errors in Making Decisions

(continued)

- **Type II Error**
  - Fail to reject a false null hypothesis

The probability of Type II Error is denoted by β.

Outcomes and Probabilities

Possible Hypothesis Test Outcomes

	Actual Situation	
Decision	H_0 True	H_0 False
Do Not Reject **H_0**	**No Error** **(1 - α)**	**Type II Error** **(β)**
Reject **H_0**	**Type I Error** **(α)**	**No Error** **(1 - β)**

Outcome
(Probability)

Criminal Trial

``Innocent until proven guilty'' → H_0: not guilty

	Actual Situation	
Verdict	H_0: not guilty	H_1: guilty
Not Guilty	**No Error** **(1 - α)**	**Type II Error** **(β)**
Guilty	**Type I Error** **(α)**	**No Error** **(1 - β)**

Outcome (Probability)

``Beyond a reasonable doubt'' → α

Presidential election

p = population fraction of Clinton voters

$H_0 : p \leq 0.5$ (Trump wins)

Outcome (Probability)		Actual Situation	
	Decision	H_0: $p \leq 0.5$	H_1: $p > 0.5$
	Do Not Reject H_0	**No Error** **($1 - \alpha$)**	**Type II Error** **(β)**
	Reject H_0	**Type I Error** **(α)**	**No Error** **($1 - \beta$)**

Example of Type I error

Using the survey of voters from Florida, the null hypothesis of $H_0: p \leq 0.5$ was rejected at the significance level of $\alpha = 0.20$. This suggests that Clinton would won Florida.

In reality, Florida was won by Donald Trump.

Type I error happens with the probability $\alpha = 0.2$.

Example of Type II error

Using the survey of voters from Minnesota, the null hypothesis of $H_0: p \leq 0.5$ was not rejected at the significance level of $\alpha = 0.05$.

This suggests that there is not strong enough evidence that Clinton would win Minnesota. In other words, survey evidence was inconclusive.

In reality, Minnesota was won by Hilary Clinton.

Type I & II Error Relationship

- Type I and Type II errors can not happen at the same time
  - Type I error can only occur if H_0 is true
  - Type II error can only occur if H_0 is false

If Type I error probability (α) ⬆, then

Type II error probability (β) ⬇

Type I & II Error in criminal trial

- Criminal Trial:

  α ⬆ → less evidence is required to give guilty verdict. More innocent person will go to jail with false guilty verdict by mistake.

  β ⬇ → convicting criminals is easier. Less criminals will be set free with non-guilty verdict by mistake.

In criminal trial, α is set to a small value.

Factors Affecting Type II Error

- All else equal,
  - β ↑ when the difference between hypothesized parameter and its true value ↓
  - β ↑ when α ↓
  - β ↑ when σ ↑
  - β ↑ when n ↓

Power of the Test

- The power of a test is the probability of correctly rejecting a false null hypothesis:

$$\text{Power} = P(\text{Reject } H_0 \mid H_1 \text{ is true})$$
$$= 1 - \beta$$

  - Power of the test increases as the sample size increases

Test of Hypothesis for the Mean (σ Known)

9.2

- Convert sample result ($\bar{x}$) to a z value

Hypothesis Tests for μ

σ Known

σ Unknown

Consider the test

$$H_0: \mu = \mu_0$$

$$H_1: \mu > \mu_0$$

(Assume the population is normal)

The decision rule is:

$$\text{Reject } H_0 \text{ if } z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \geq z_\alpha$$

Decision Rule

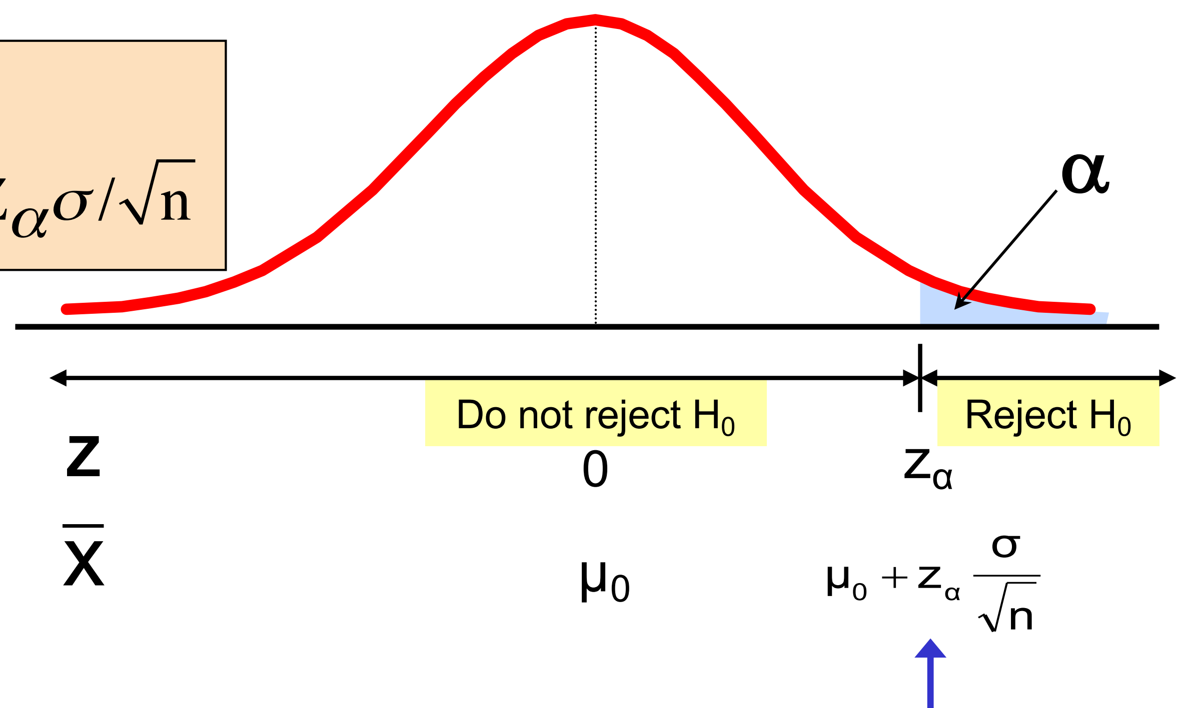
$H_0: \mu = \mu_0$

$H_1: \mu > \mu_0$

$$\text{Reject } H_0 \text{ if } z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \geq z_\alpha$$

Alternate rule:

$$\text{Reject } H_0 \text{ if } \bar{x} \geq \mu_0 + Z_\alpha \sigma / \sqrt{n}$$



Critical value C

p-Value Approach to Testing

- p-value: Probability of obtaining a test statistic more extreme (≤ or ≥) than the observed sample value given H_0 is true
  - Also called observed level of significance
  - Smallest value of α for which H_0 can be rejected

p-Value Approach to Testing

(continued)

- Convert sample result (e.g., $\bar{x}$) to test statistic (e.g., z statistic)

- Obtain the p-value
  - For an upper tail test:

$$\text{p-value} = P\left(z \geq \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}, \text{ given that } H_0 \text{ is true}\right)$$

$$= P\left(z \geq \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \mid \mu = \mu_0\right)$$

- Decision rule: compare the p-value to α
  - If p-value $\leq \alpha$, reject H_0
  - If p-value $> \alpha$, do not reject H_0

Upper-Tail Z Test for Mean (σ Known)

A phone industry manager thinks that customer monthly cell phone bill have increased, and now average over $52 per month. The company wishes to test this claim. (Assume $\sigma = 10$ is known)

Form hypothesis test:

H_0: $\mu \leq 52$ the average is not over $52 per month

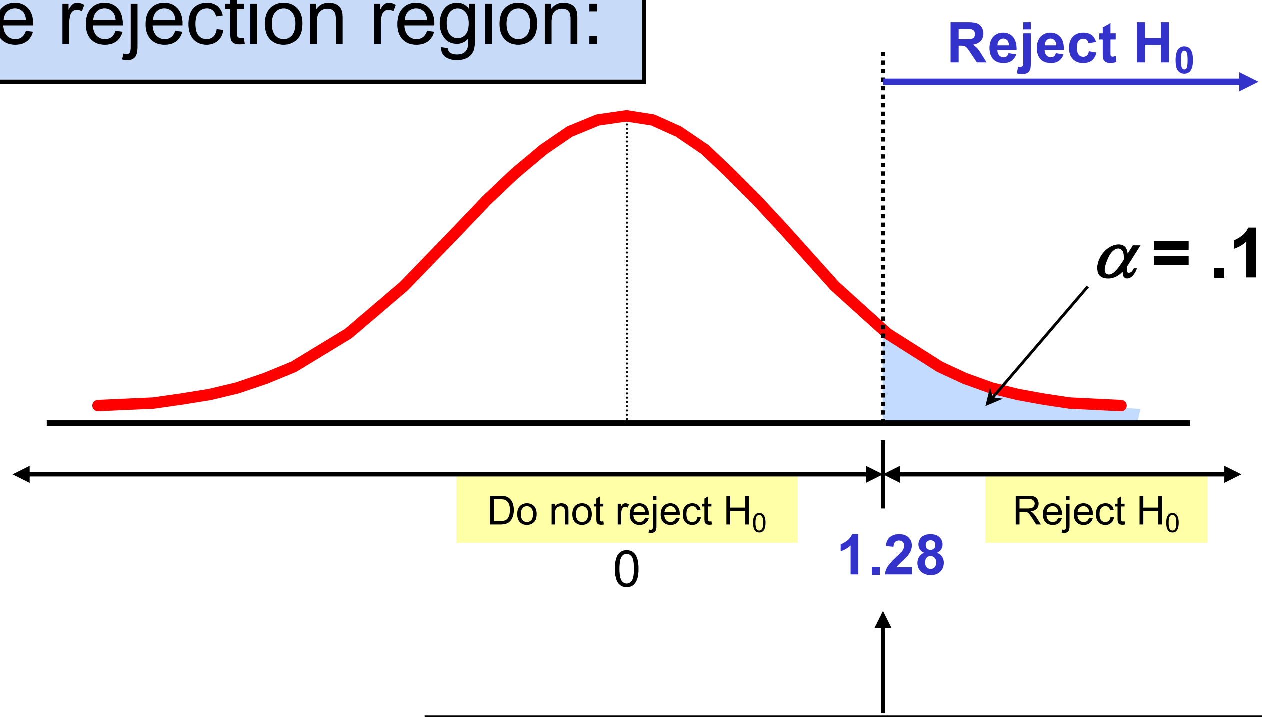
H_1: $\mu > 52$ the average is greater than $52 per month (i.e., sufficient evidence exists to support the manager's claim)

Example: Find Rejection Region

(continued)

- Suppose that α = .10 is chosen for this test

Find the rejection region:



$$\text{Reject } H_0 \text{ if } z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \geq 1.28$$

Example: Sample Results

(continued)

Obtain sample and compute the test statistic

Suppose a sample is taken with the following results: $n = 64$, $\bar{x} = 53.1$ ($\sigma = 10$ was assumed known)

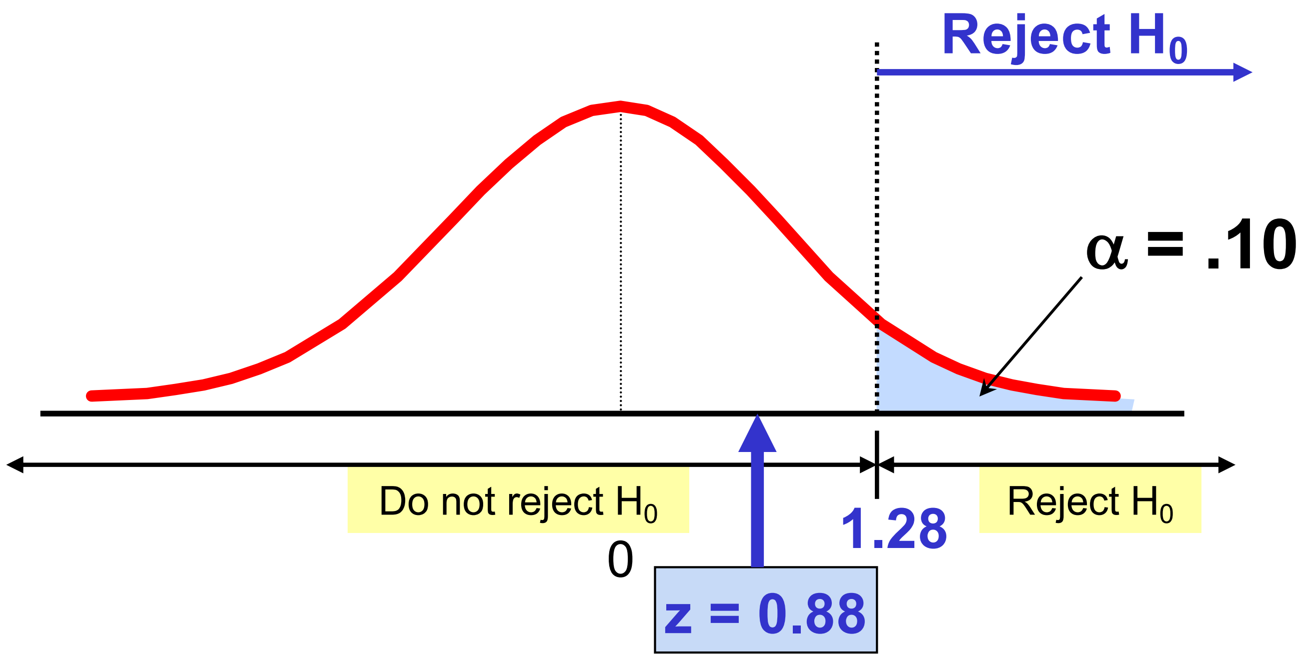
- Using the sample results,

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{53.1 - 52}{\frac{10}{\sqrt{64}}} = 0.88$$

Example: Decision

(continued)

Reach a decision and interpret the result:



Do not reject H_0 since $z = 0.88 < 1.28$

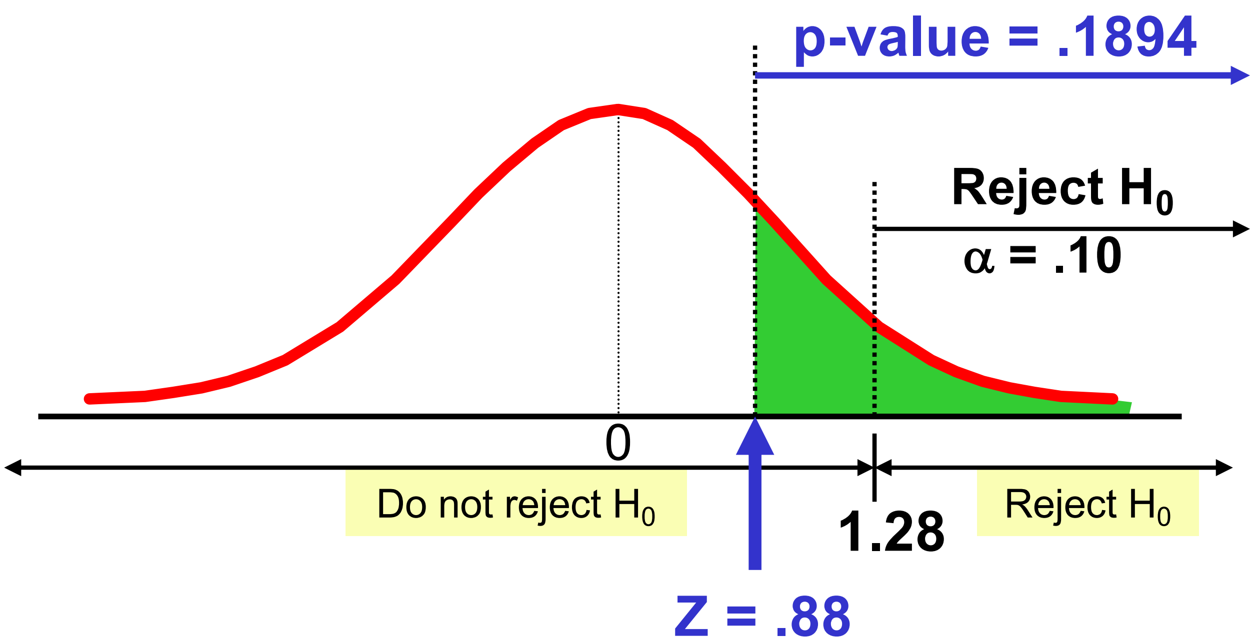
i.e.: there is not sufficient evidence that the mean bill is over $52

Example: p-Value Solution

(continued)

Calculate the p-value and compare to α

(assuming that μ = 52.0)



$$P(\bar{x} \geq 53.1 \mid \mu = 52.0)$$

$$= P\left(z \geq \frac{53.1 - 52.0}{10/\sqrt{64}}\right)$$

$$= P(z \geq 0.88) = 1 - .8106$$

$$= .1894$$

Do not reject H_0 since p-value = .1894 > α = .10

Worksheet Question 2

- On Oct 24 of 2016, the survey was conducted in Florida after the final presidential debate.
- p = population fraction of Clinton supporters
- Among $n = 1166$ likely registered voters, there are 602 Clinton voters and 564 Trump voters so that $\hat{p} = 0.516$
- **What is the p-value of testing** $H_0: p \leq 0.5$?

Answer

The distribution of $\hat{p}$ when $p = 0.5$ and $n = 1166$:

$$\frac{\hat{p} - 0.5}{\sqrt{0.5(1-0.5)/1166}} = \frac{\hat{p} - 0.5}{0.015} \sim N(0,1)$$

The realized value is $\hat{p} = 0.516$.

$$P\left(Z > \frac{0.516 - 0.5}{0.015}\right) = P(Z > 1.07) = 0.1423$$

One-Tail Tests

- In many cases, the alternative hypothesis focuses on one particular direction

$H_0: \mu \leq 3$

$H_1: \mu > 3$

→ This is an upper-tail test since the alternative hypothesis is focused on the upper tail above the mean of 3

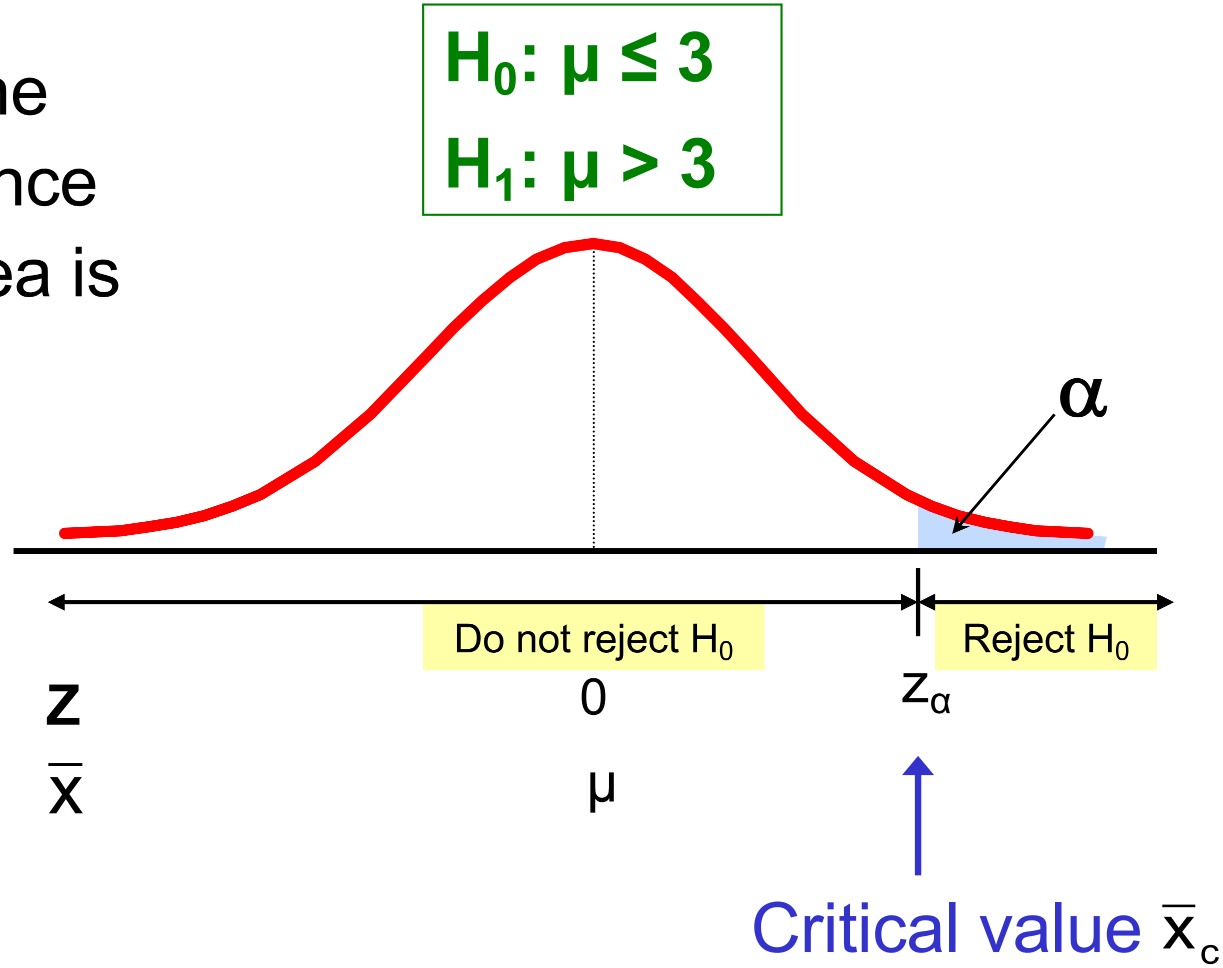
$H_0: \mu \geq 3$

$H_1: \mu < 3$

→ This is a lower-tail test since the alternative hypothesis is focused on the lower tail below the mean of 3

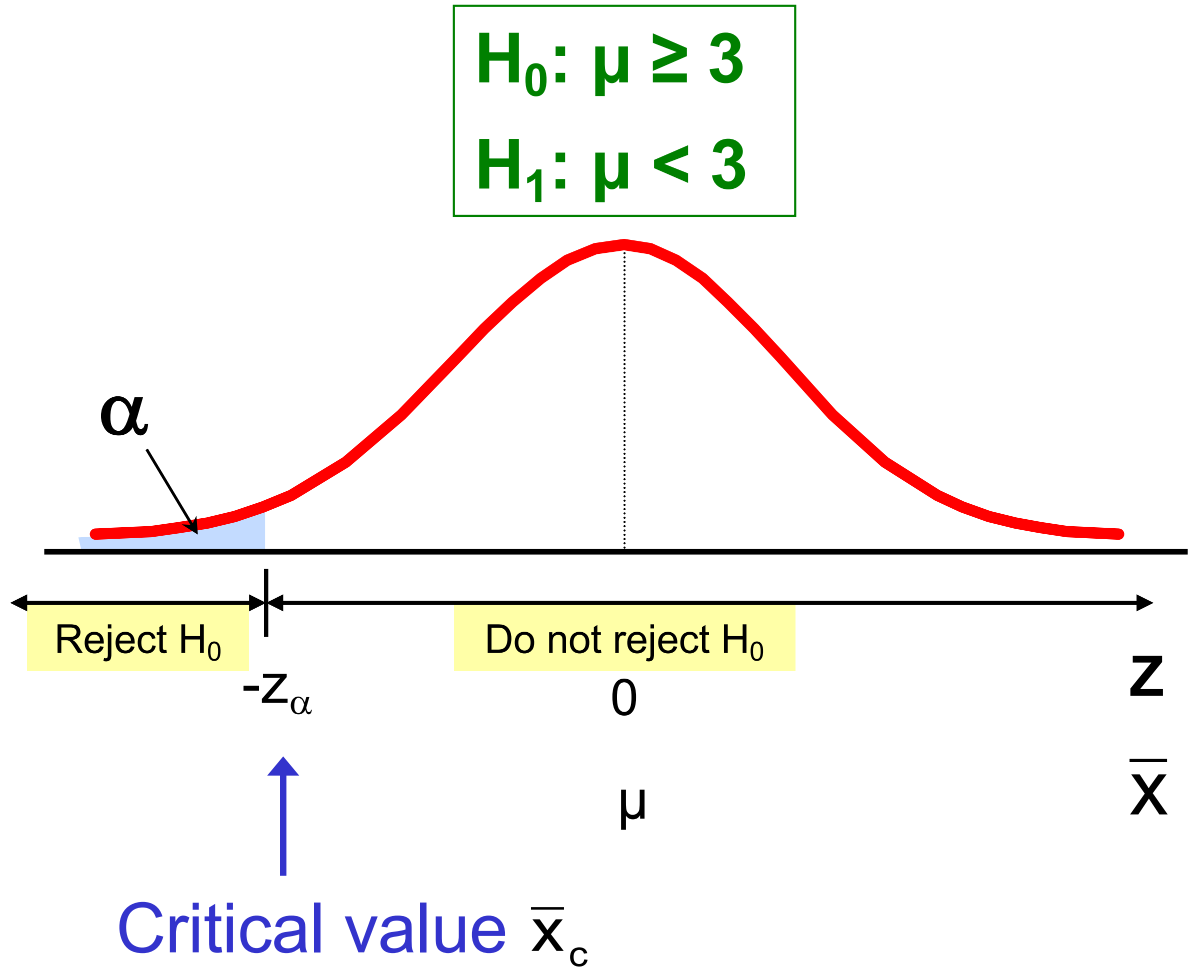
Upper-Tail Tests

- There is only one critical value, since the rejection area is in only one tail



Lower-Tail Tests

- There is only one critical value, since the rejection area is in only one tail

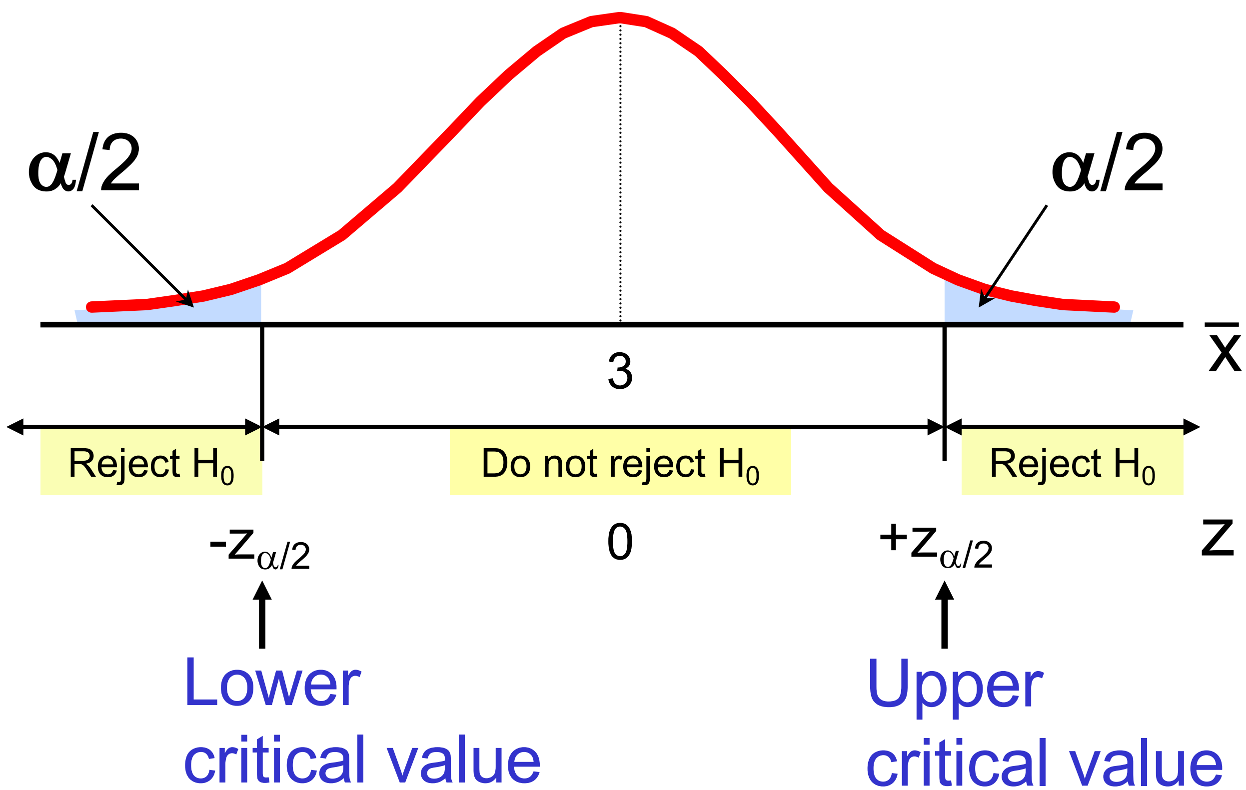


Two-Tail Tests

- In some settings, the alternative hypothesis does not specify a unique direction

H_0: μ = 3
H_1: μ ≠ 3

- There are two critical values, defining the two regions of rejection



Confidence Interval and Hypothesis Test

Suppose the 95 percent confidence interval for μ is given by $[3.5, 6.9]$, i.e.,

$$P(3.5 \le \mu \le 6.9) = 0.95$$

Then, μ is likely to lie above 3.5, providing the evidence against $H_0: \mu = 3$ at $\alpha = 0.05$.

The 95 percent confidence interval for μ

$\leftrightarrow$ Two-tail test of $H_0: \mu = 3$ at $\alpha = 0.05$.

Clicker Question 9.4

Suppose that the 95 percent confidence interval for μ is given by $[3.5, 6.9]$, i.e.,

$$P(3.5 \leq \mu \leq 6.9) = 0.95$$

Then,

A). We always reject $H_0: \mu = 3$ (two tail test) both at $\alpha = 0.05$ and at $\alpha = 0.01$

B). We always reject $H_0: \mu = 3$ (two tail test) both at $\alpha = 0.05$ and at $\alpha = 0.10$

Clicker Question 9.5

The null hypothesis H_0: $\mu \leq 3$ (upper tail test) at significant level $\alpha = 0.05$ is rejected if and only if

A). The 95 percent confidence interval for μ lies above 3.

B). The 90 percent confidence interval for μ lies above 3.

C). The 97.5 percent confidence interval for μ lies above 3.

Two-Tail Hypothesis Test

Test the claim that the true mean # of TV sets in US homes is not equal to 3. (Assume $\sigma = 0.8$)

- State the appropriate null and alternative hypotheses
  - H_0: $\mu = 3$, H_1: $\mu \neq 3$ (This is a two tailed test)
- Specify the desired level of significance
  - Suppose that $\alpha = .05$ is chosen for this test
- Choose a sample size
  - Suppose a sample of size n = 100 is selected

Two-Tail Hypothesis Test

(continued)

- Determine the appropriate technique
  - σ is known so this is a z test
- Set up the critical values
  - For $\alpha = .05$ the critical z values are ±1.96
- Collect the data and compute the test statistic
  - Suppose the sample results are

    $n = 100$, $\bar{x} = 2.84$ ($\sigma = 0.8$ is assumed known)

So the test statistic is:

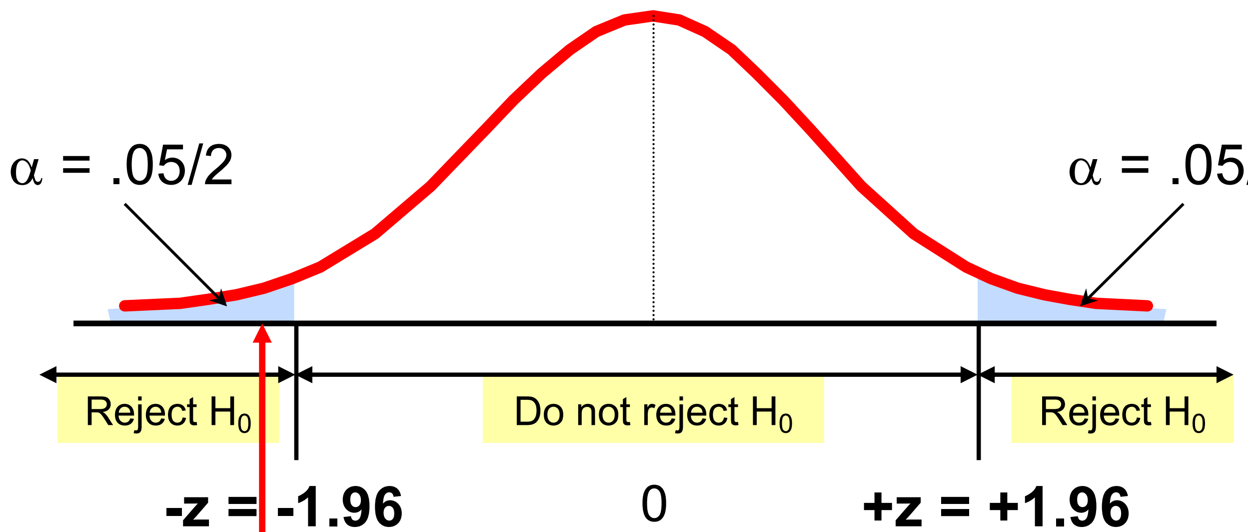
$$z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{2.84 - 3}{\frac{0.8}{\sqrt{100}}} = \frac{-.16}{.08} = -2.0$$

Two-Tail Hypothesis Test

(continued)

- Is the test statistic in the rejection region?

Reject H_0 if $z \leq -1.96$ or $z \geq 1.96$; otherwise do not reject H_0

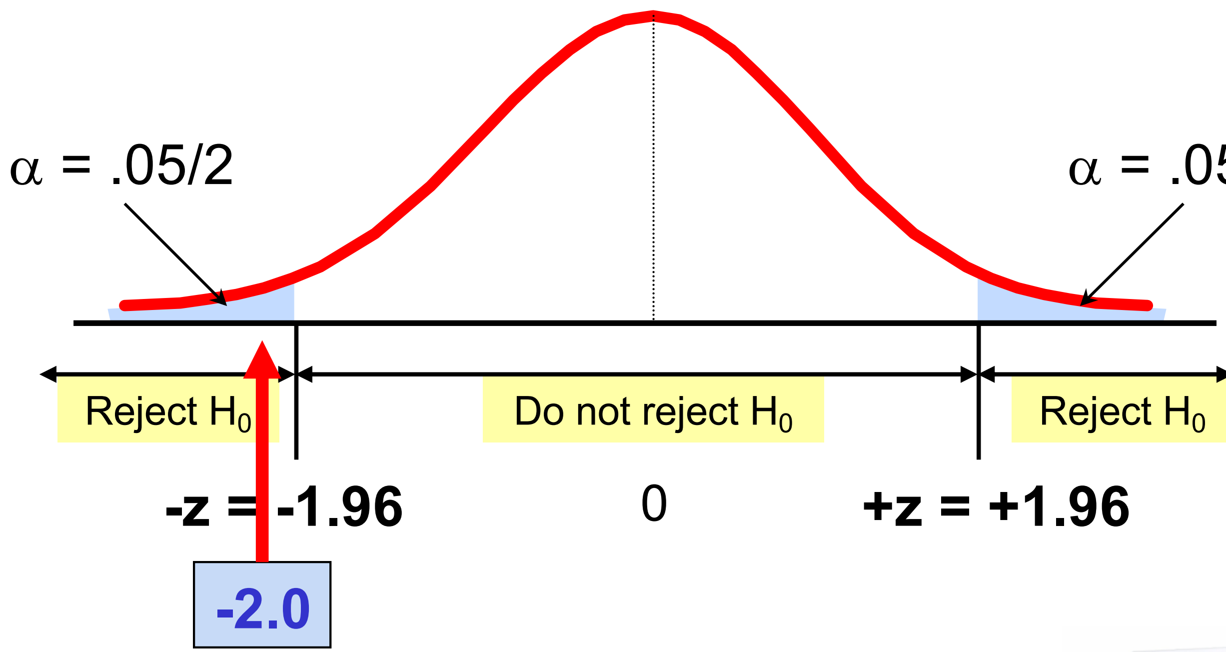


Here, $z = -2.0 < -1.96$, so the test statistic is in the rejection region

Two-Tail Hypothesis Test

(continued)

- Reach a decision and interpret the result



Since $z = -2.0 < -1.96$, we reject the null hypothesis and conclude that there is sufficient evidence that the mean number of TVs in US homes is not equal to 3

Confidence Interval

- In this example, the 95 percent confidence interval for μ is given by $[3.5, 6.9]$, i.e.,

$$2.84 \pm 1.96 \times 0.08 = [2.683\,, 2.997]$$

μ = 3 is not in the 95 percent confidence interval.

↔ We reject $H_0: \mu = 3$ (two tail) at $\alpha = 0.05$.

↔ We reject $H_0: \mu \leq 3$ (one tail) at $\alpha = 0.025$.

p-Value for Two-tail Test

- **Example:** How likely is it to see a sample mean of 2.84 (or something further from the mean, in either direction) if the true mean is μ = 3.0?

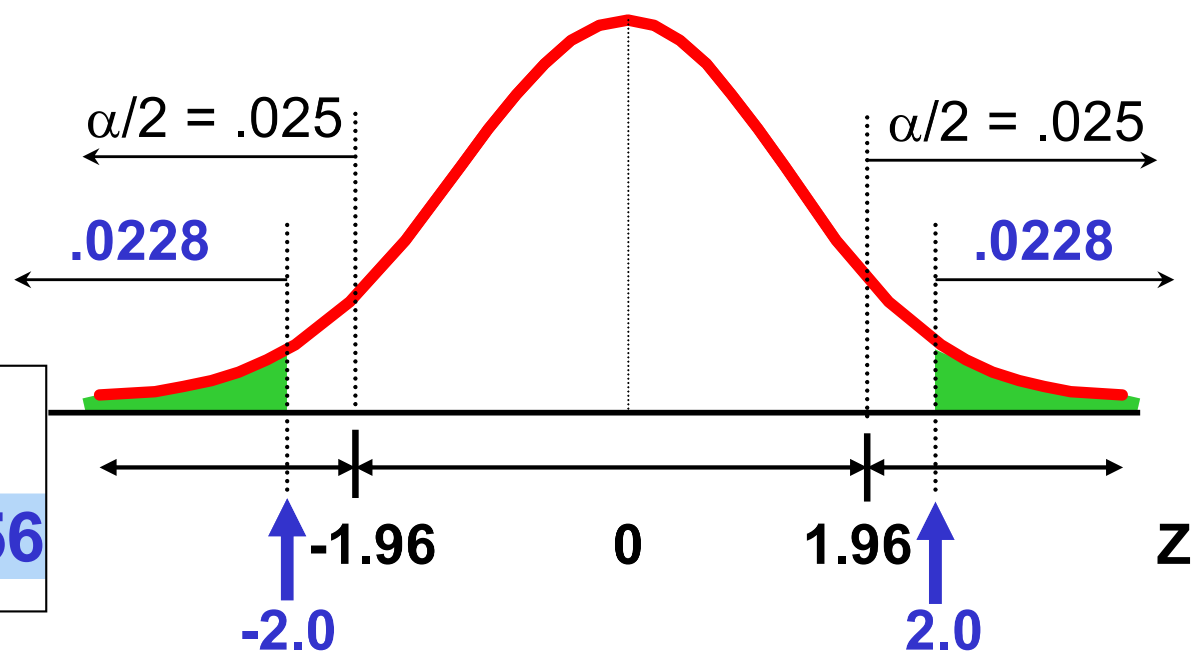
$\bar{x}$ = 2.84 is translated to a z score of z = -2.0

$$P(z < -2.0) = .0228$$

$$P(z > 2.0) = .0228$$

p-value

= .0228 + .0228 = .0456



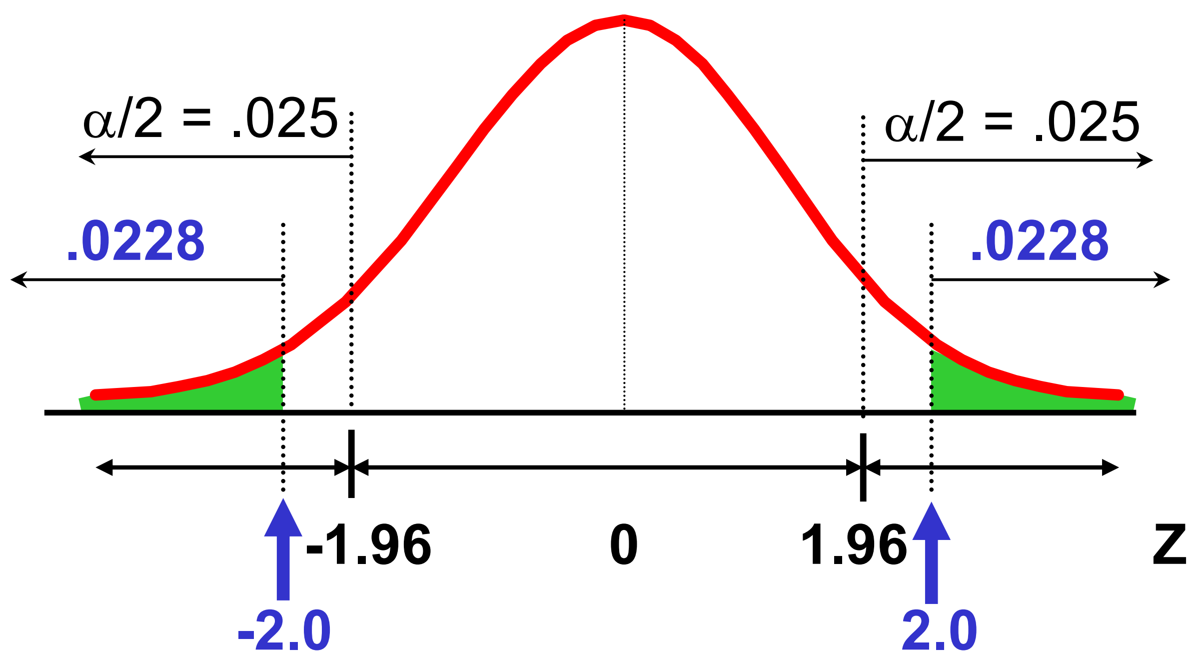
p-Value for Two-tail Test

(continued)

- Compare the p-value with α
  - If p-value $\leq$ α , reject H_0
  - If p-value **>** α , do not reject H_0

Here: p-value = .0456
α = .05

Since .0456 < .05, we reject the null hypothesis



t Test of Hypothesis (σ Unknown)

(continued)

- For a two-tailed test:

Consider the test

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu \neq \mu_0$$

(Assume the population is normal, and the population variance is unknown)

The decision rule is:

$$\text{Reject } H_0 \text{ if } \quad t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} \leq -t_{n\text{-}1,\ \alpha/2} \quad \text{or if} \quad t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} \leq t_{n\text{-}1,\ \alpha/2}$$

t Test of Hypothesis (σ Unknown)

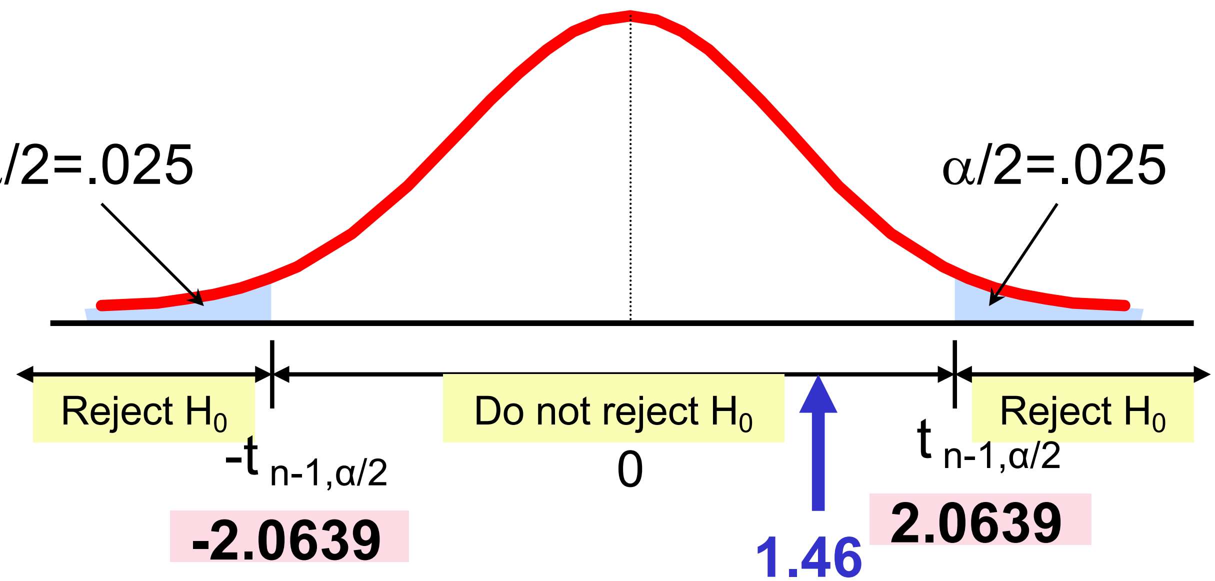
The average cost of a hotel room in Chicago is said to be \$168 per night. A random sample of 25 hotels resulted in $\bar{x}$ = \$172.50 and s = \$15.40. Test at the α = 0.05 level.

(Assume the population distribution is normal)

H_0: μ = 168
H_1: $\mu \neq 168$

Solution: Two-Tail Test

H_0: $\mu = 168$
H_1: $\mu \neq 168$



- $\alpha = 0.05$
- $n = 25$
- σ is unknown, so use a **t statistic**
- Critical Value:

$t_{24\,,\,.025} = \pm\ 2.0639$

$$t_{n-1} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{172.50 - 168}{\frac{15.40}{\sqrt{25}}} = 1.46$$

Do not reject H_0: not sufficient evidence that true mean cost is different than \$168

Power of the Test

- Recall the possible hypothesis test outcomes:

	Actual Situation	
Decision	H_0 True	H_0 False
Do Not Reject H_0	**No error** **(1 - α)**	**Type II Error** **(β)**
Reject H_0	**Type I Error** **(α)**	**No Error** **(1 - β)**

Key:
Outcome
(Probability)

- β denotes the probability of Type II Error
- $1 - \beta$ is defined as the power of the test

Power = $1 - \beta$ = the probability that a false null hypothesis is rejected

Type II Error (β)

Assume the population is normal and the population variance is known. Consider the test

$$H_0 : \mu \geq \mu_0 = 52$$
$$H_1 : \mu < \mu_0 = 52$$

The decision rule is:

$$\text{Reject } H_0 \text{ if } z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \leq z_\alpha$$

or

$$\text{Reject } H_0 \text{ if } \bar{x} = \bar{x}_c \leq \mu_0 + z_\alpha \sigma / \sqrt{n}$$

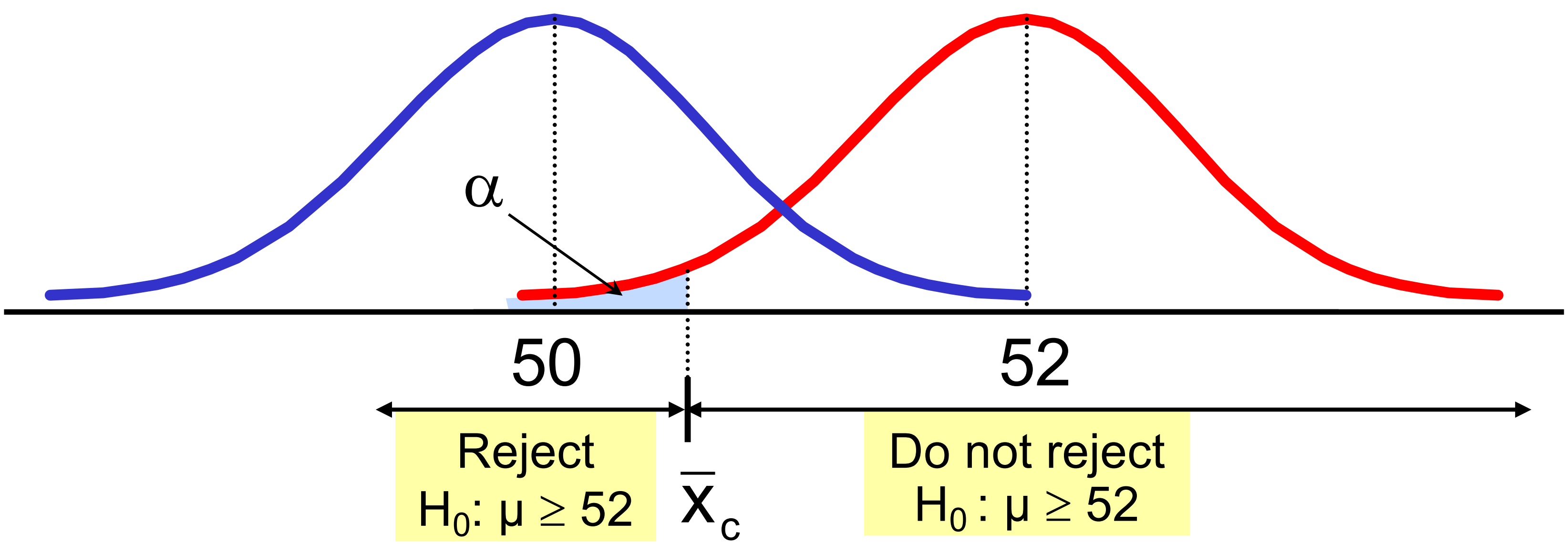
If the null hypothesis is false and the true mean is μ*, then the probability of type II error is

$$\beta = P(\bar{x} > \bar{x}_c \mid \mu = \mu^*) = P\left(z > \frac{\bar{x}_c - \mu^*}{\sigma / \sqrt{n}} \right)$$

Type II Error Example

- Type II error is the probability of failing to reject a false H_0

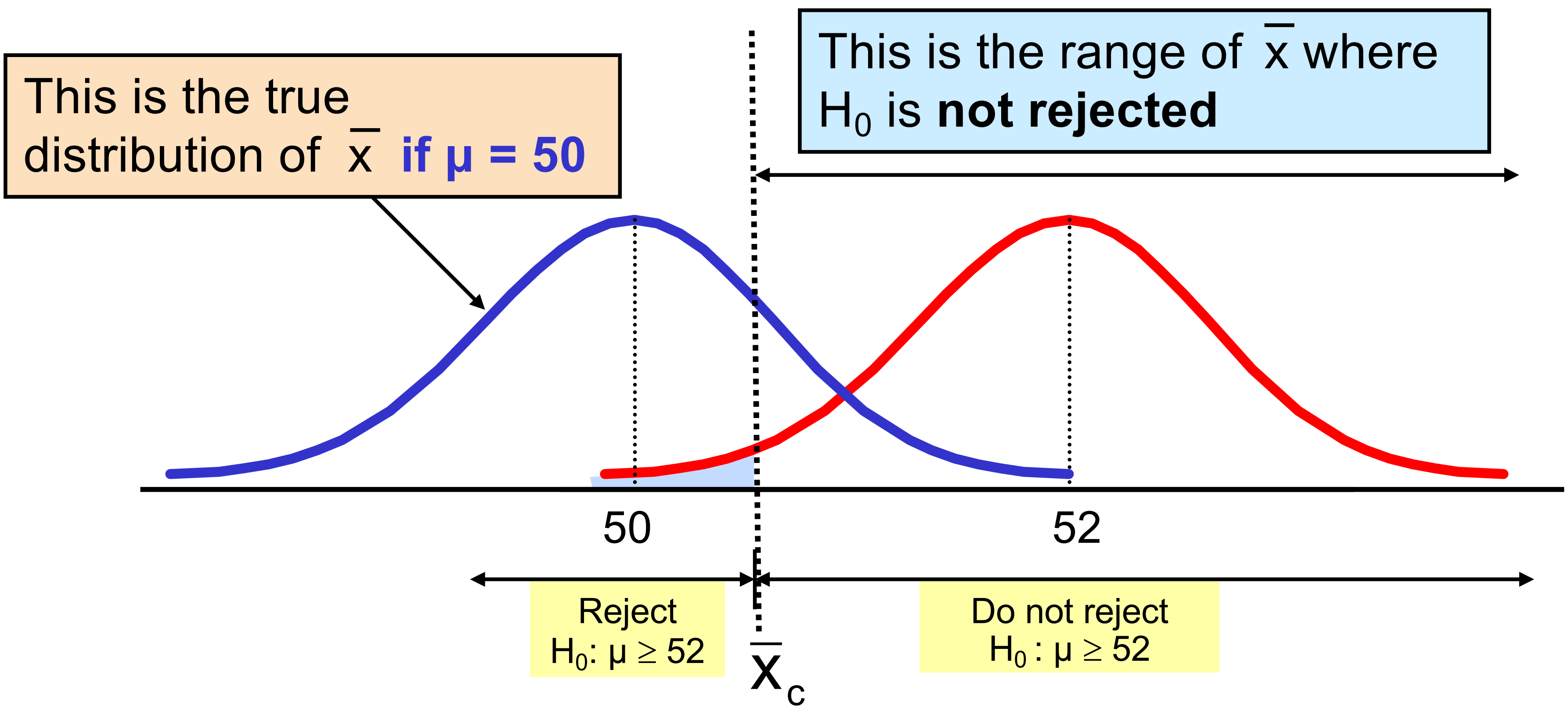
Suppose we fail to reject H_0: $\mu \geq 52$ when in fact the true mean is $\mu^* = 50$



Type II Error Example

(continued)

- Suppose we do not reject H_0: $\mu \geq 52$ when in fact the true mean is $\mu^* = 50$

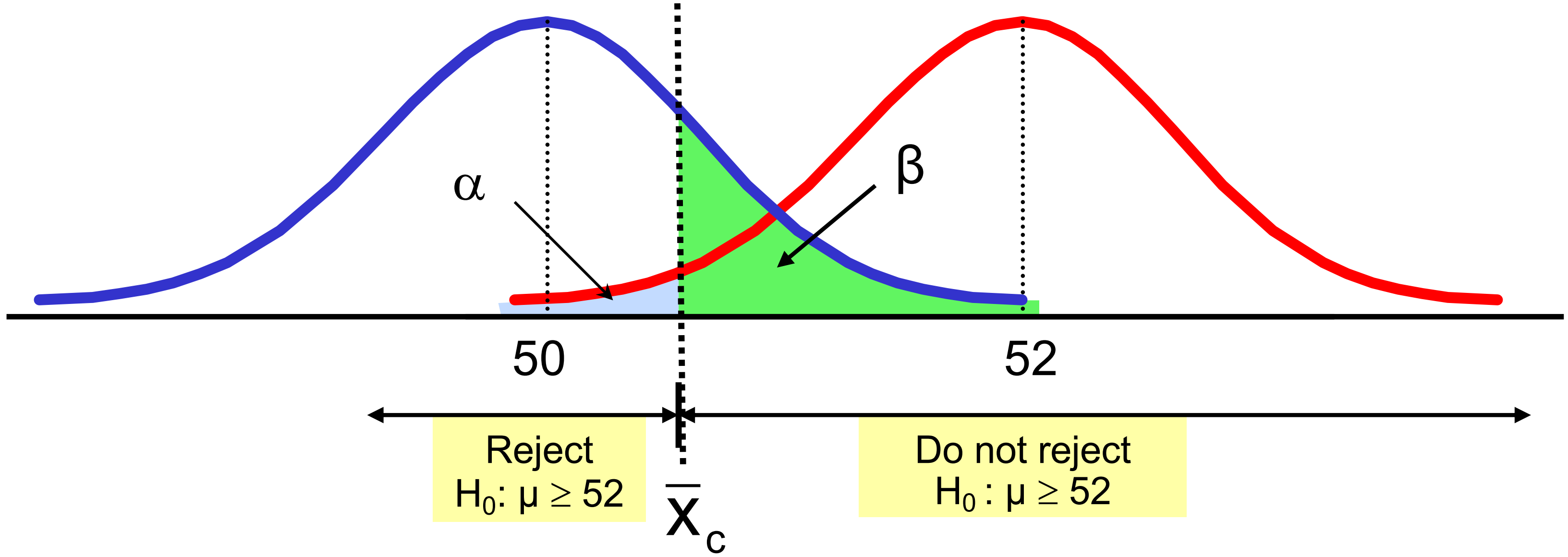


Type II Error Example

(continued)

- Suppose we do not reject H_0: $\mu \geq 52$ when in fact the true mean is $\mu^* = 50$

Here, $\beta = P(\bar{x} \geq \bar{x}_c)$ if $\mu^* = 50$



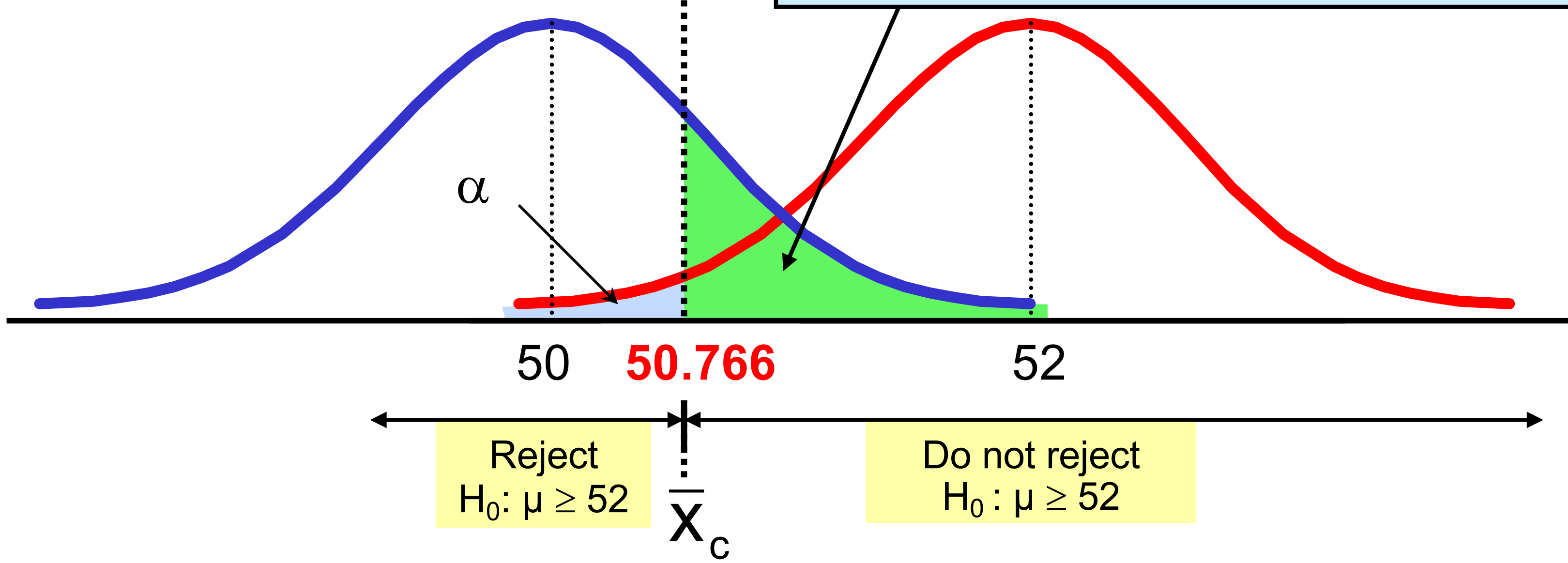
Calculating Type II Error (β)

- Suppose n = 64 , σ = 6 , and α = .05

$$\bar{x}_c = \mu_0 - z_\alpha \frac{\sigma}{\sqrt{n}} = 52 - 1.645 \frac{6}{\sqrt{64}} = 50.766$$

(for $H_0 : \mu \geq 52$)

So $\beta = P(\bar{x} \geq 50.766)$ if $\mu^* = 50$

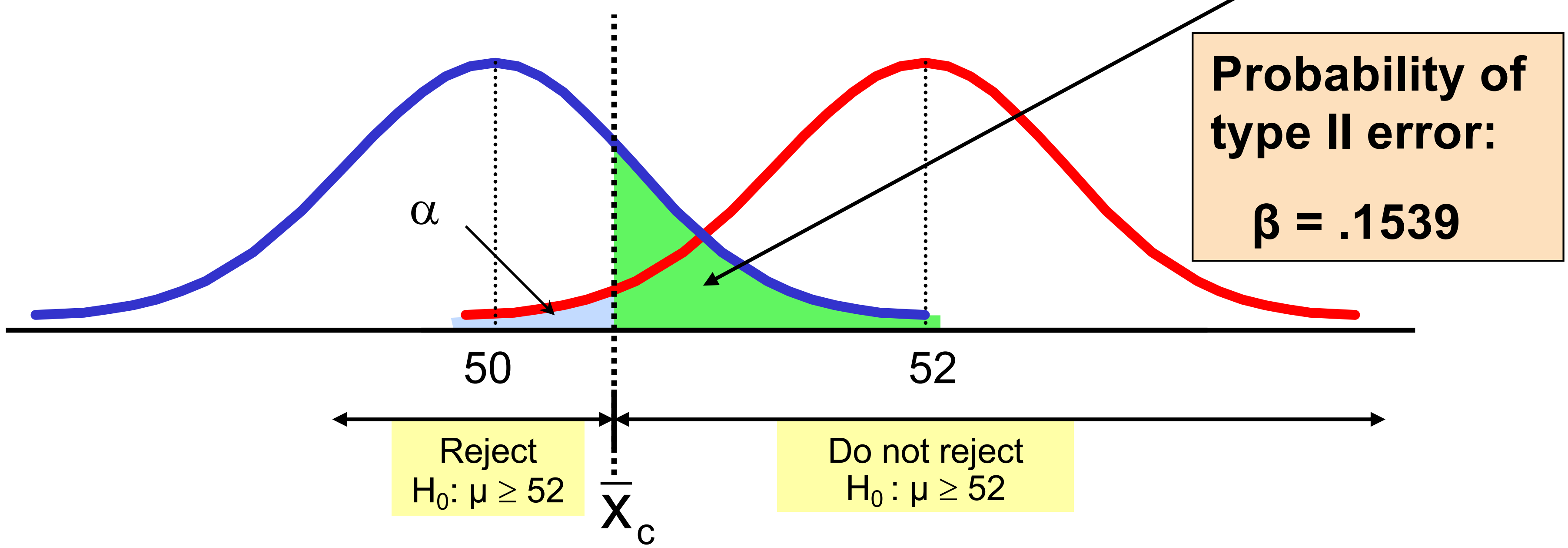


Calculating Type II Error (β)

(continued)

- Suppose n = 64 , σ = 6 , and α = .05

$$P(\bar{x} \geq 50.766 \mid \mu^* = 50) = P\left(z \geq \frac{50.766 - 50}{6/\sqrt{64}} \right) = P(z \geq 1.02) = .5 - .3461 = .1539$$



Power of the Test Example

If the true mean is $\mu^* = 50$,

- The probability of Type II Error = $\beta = 0.1539$
- The power of the test = $1 - \beta = 1 - 0.1539 = 0.8461$

Key:
Outcome
(Probability)

	Actual Situation	
Decision	H_0 True	H_0 False
Do Not Reject H_0	**No error** **$1 - \alpha = 0.95$**	**Type II Error** **$\beta = 0.1539$**
Reject H_0	**Type I Error** **$\alpha = 0.05$**	**No Error** **$1 - \beta = 0.8461$**

(The value of β and the power will be different for each μ^*)

How to compute the power

- Step 1: Find the distribution of $\bar{x}$ if the null hypothesis of $\mu = 52$ is true and compute the critical value and rejection region.
- Step 2: Find the distribution of $\bar{x}$ if $\mu^* = 52$ (i.e., the true value of μ).
- Step 3: Power is computed as the probability of $\bar{x}$ falls into the rejection region under the distribution of $\bar{x}$ when $\mu^* = 52$.

Worksheet Question 3

- p = population fraction of Clinton supporters.
- Suppose we test the null hypothesis of $H_0: p \leq 0.5$ at significant level $\alpha = 0.10$ using the random sample of $n = 921$ voters.
- In Virginia, among voters who support either Clinton or Trump, 52.9 percent of voters voted for Clinton so that $p = 0.529$.
- What is the power of the test?

How to compute the power

- Step 1: Find the distribution of $\hat{p}$ if the null hypothesis of $p = 0.5$ is true and compute the critical value and rejection region.
- Step 2: Find the distribution of $\hat{p}$ if $p = 0.529$ (i.e., the true value of p).
- Step 3: Power is computed as the probability of $\hat{p}$ falls into the rejection region under the distribution of $\hat{p}$ when $p = 0.529$.

Worksheet Question 3

- The rejection region is given by $[0.521, \infty)$
- In survey of Oct 30, 2017, $\hat{p} = 0.533.$
- Power is computed as

$$P(\hat{p} > 0.521 \mid p = 0.529)$$
$$= P\left(\frac{\hat{p} - 0.529}{0.0164} > \frac{0.521 - 0.529}{0.0164}\right) = P(Z > -0.488)$$
$$= 0.6879$$

- $H_0\colon p \le 0.5$ will be correctly rejected with 0.6879 probability.

Hypothesis Tests of one Population Variance

(continued)

The test statistic for hypothesis tests about one population variance is

$$\chi^2_{n-1} = \frac{(n-1)s^2}{\sigma_0^2}$$

9.6

Hypothesis Tests of one Population Variance

- Goal: Test hypotheses about the population variance, σ^2

  - If the population is normally distributed,

$$\chi^2_{n-1} = \frac{(n-1)s^2}{\sigma^2}$$

has a chi-square distribution with (n – 1) degrees of freedom

Decision Rules: Variance

Population variance

Lower-tail test:

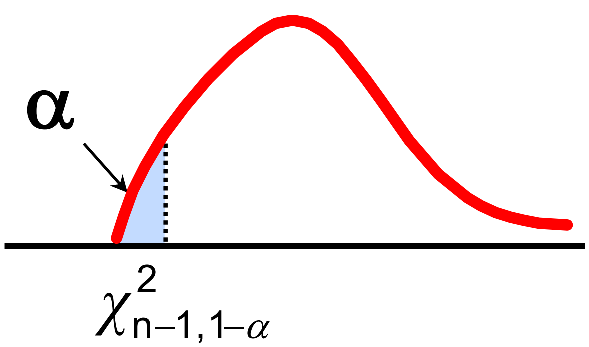
$H_0: \sigma^2 \geq \sigma_0^2$
$H_1: \sigma^2 < \sigma_0^2$

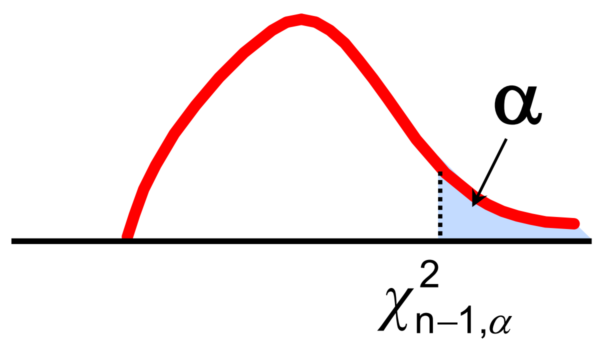
Upper-tail test:

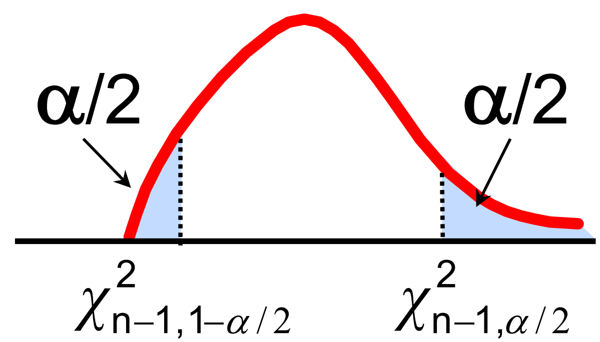
$H_0: \sigma^2 \leq \sigma_0^2$
$H_1: \sigma^2 > \sigma_0^2$

Two-tail test:

$H_0: \sigma^2 = \sigma_0^2$
$H_1: \sigma^2 \neq \sigma_0^2$







Reject H_0 if

$$\chi^2_{n-1} \leq \chi^2_{n-1,1-\alpha}$$

Reject H_0 if

$$\chi^2_{n-1} \geq \chi^2_{n-1,\alpha}$$

Reject H_0 if

$$\chi^2_{n-1} \geq \chi^2_{n-1,\alpha/2}$$

or

$$\chi^2_{n-1} \leq \chi^2_{n-1,1-\alpha/2}$$