

## Midterm Exam

1. (8 points) Given the data set that contains variables named “salary” and “roe,” what Stata command computes the correlation coefficient as in the following output?

```
(obs=209)

      |      salary      roe
-----+-----
salary |      1.0000
      |
      roe |      0.1148      1.0000
```

2. Multiple Choice Questions (No Explanation Necessary):

(a) (8 points) Suppose that both  $X$  and  $Y$  are random variables and are not constants. Which of the followings is generally **False**?

- A)  $E[Y|X] = E[Y]$  if  $X$  and  $Y$  are independent.    B)  $E[Y|X]$  is always a constant.  
C)  $E[Y|X]$  is not always a constant but generally a random variable.

(b) (8 points) Suppose that both  $X$  and  $Y$  are random variables and define  $W = (X - E(X))/\sqrt{Var(X)}$  and  $Z = (Y - E(Y))/\sqrt{Var(Y)}$ . Which of the followings is generally **False**?

- A)  $Cov(W, Z) = Corr(X, Y)$     B)  $Var(W) = 1$     C)  $E[WZ] = Corr(X, Y)$     D)  $Var(WZ) = Var(XY)$ .

(c) (8 points) If A and B are independent events with  $P(A) = 0.30$  and  $P(B) = 0.40$ , then the probability that A occurs or B occurs or both occur is:

- A) 0.12    B) 0.58    C) 0.70    D) 0.82.

3. (10 points) In one year, the average stock price of Google Inc. was \$560 with the standard deviation equal to \$30. Using the empirical rule, it can be estimated that approximately 95 % of the stock price of Google Inc. will be in what interval?

4. (10 points) Consider the joint probability distribution of  $(X, Y)$ :

		X	
		0	1
Y	0	0.2	0.4
	1	0.3	0.1

Compute the covariance between  $X$  and  $Y$ .

5. (10 points) A review of the personnel records of a small corporation has revealed the following information about the number of sick days taken per year and the corresponding probabilities.

Number of Sick Days	0	1	2	3	4	5
Probability	0.05	0.22	0.31	0.27	0.13	0.02

Let  $A$  be the event that an employee takes more than 2 sick days (i.e., Number of Sick Days  $\geq 3$ ). Compute the probability of event  $A$ .

6. (10 points) Given the following table, what is the probability that a randomly selected person is female conditional on the selected person does not support US policy in Iraq?

	US Policy in Iraq		
	Support	Doesn't Support	Row Total
Female	0.2438	0.2862	0.53
Male	0.3762	0.0938	0.47
Column Total	0.62	0.38	1.00

7. Let  $X$  and  $Y$  be two discrete random variables. The set of possible values for  $X$  is  $\{x_1, \dots, x_n\}$ ; and the set of possible values for  $Y$  is  $\{y_1, \dots, y_m\}$ . The joint function of  $X$  and  $Y$  is given by  $p_{ij}^{X,Y} = P(X = x_i, Y = y_j)$  for  $i = 1, \dots, n; j = 1, \dots, m$ . The marginal probability function of  $X$  is  $p_i^X = P(X = x_i) = \sum_{j=1}^m p_{ij}^{X,Y}$  for  $i = 1, \dots, n$ , and the marginal probability function of  $Y$  is  $p_j^Y = P(Y = y_j) = \sum_{i=1}^n p_{ij}^{X,Y}$  for  $j = 1, \dots, m$ .

Prove the following results for general  $n$  and  $m$ .

- (a) (10 points) Prove that  $Cov((X - E(X)), Y) = Cov(X, Y)$ .

(b) (10 points) Prove that  $E(XY) = E(X)E(Y)$  if  $X$  and  $Y$  are independent using the summation operator together with notations  $p_{ij}^{X,Y}$ ,  $p_i^X$ , and  $p_j^Y$  defined above.

(c) (8 points) Multiple Choice Questions (No Explanation Necessary). Which of the followings is

**True:**

A)  $E[Y|X = x_i] = \sum_{j=1}^m y_j \frac{p_{ij}^{X,Y}}{p_i^X}$       B)  $E[Y|X = x_i] = \sum_{j=1}^m y_j p_{ij}^{X,Y}$   
C)  $E[Y|X = x_i] = \sum_{j=1}^m y_j p_j^Y p_i^X$       D)  $E[Y|X = x_i] = \sum_{j=1}^m y_j \frac{p_{ij}^{X,Y}}{p_j^Y}$ .