Economics 325–003 **Term I 2015**

Introduction to Empirical Economics **Hiro Kasahara**

Midterm Exam

1. (8 points) The data set contains variables "salary" and "roe". What is a Stata command that will reproduce the following Stata output? (Please write a whole line of command that produces the output.)

Variable	Obs	Mean	Std. Dev.	Min	Max
salary	209	1281.12	1372.345	223	14822
roe	209	17.18421	8.518509	.5	56.3

Answer: "summarize salary roe"

2. (8 points) Suppose that $P(A) = 0.30$, $P(B|A) = 0.60$, and $P(B|\bar{A}) = 0.60$, where $\bar{A}$ and $\bar{B}$ are complement of A and B, respectively. What is the probability of $P(\bar{A}|\bar{B})$?

Answer: Note that

$$P(\bar{A}|\bar{B}) = \frac{P(\bar{A} \cap \bar{B})}{1 - P(B)} = \frac{P(\bar{B}|\bar{A})P(\bar{A})}{1 - [P(B \cap A) + P(B \cap \bar{A})]} = \frac{(1 - P(B|\bar{A}))P(\bar{A})}{1 - [P(B|A)P(A) + P(B|\bar{A})P(\bar{A})]} = \frac{(1 - 0.6) \times 0.7}{1 - (0.6 \times 0.3 + 0.6 \times 0.7}$$

which is equal to 0.7.

3. (8 points) In one year, the average stock price of Apple Inc. was \$650 with the standard deviation equal to \$100. Using the empirical rule, it can be estimated that approximately 95 % of the stock price of Apple Inc. will be in what interval?

Answer: $650 \pm 2 \times 100$ **gives** $[450, 850]$**.**

4. In a recent survey of 300 teenagers, 60% of the teenagers indicated that they had been to a movie within the past month. 70% of those teenagers who had seen a movie also had gone out to dinner in the past month, while only 50% of the teenagers who had not seen a movie had been out to dinner in the past month. Define the random variables as follows:

X = 1 if teenager had been to movie; X = 0 otherwise

Y = 1 if teenager had been out to dinner; Y = 0 otherwise

(a) (14 points) Find the joint probability function of X and Y, the marginal distribution of X, and the marginal distribution of Y by completing empty cells in the following table.

	X = 0	X = 1	Marginal Dist. of Y
Y = 0			
Y = 1			
Marginal Dist. of X		0.60	1.00

Answer:

	X = 0	**X = 1**	**Marginal Dist. of Y**
Y = 0	**0.2**	**0.18**	**0.38**
Y = 1	**0.2**	**0.42**	**0.62**
Marginal Dist. of X	**0.40**	**0.60**	**1.00**

(b) (8 points) Compute the covariance between X and Y.

Answer: Cov$(X, Y) = E(XY) - E(X)E(Y) = 0.42 - 0.6 \times 0.62 = 0.048$**.**

(c) (8 points) Find the value of $E[X|Y = 1] - E[X|Y = 0]$.

Answer: Note that $P(X = 1|Y = 0) = P(X = 1, Y = 0)/P(Y = 0) = 0.18/0.38 = 0.4737$**. Then, because** $E[X|Y = 1] = 1 \times P(X = 1|Y = 1) + 0 \times P(X = 0|Y = 1) = P(X = 1|Y = 1)$ **and** $E[X|Y = 0] = 1 \times P(X = 1|Y = 0) + 0 \times P(X = 0|Y = 0) = P(X = 1|Y = 0)$**, we have** $E[X|Y = 1] - E[X|Y = 0] = P(X = 1|Y = 1) - P(X = 1|Y = 0) = 0.6774 - 0.4737 = 0.2037$**.**

5. Multiple Choice Questions (No Explanation Necessary):

(a) (8 point) If X and Y are independent random variables, which of the following identities is **generally false**?

A) $Cov(X, X + Y) = Var(X) + E[XY]$. B) $Cov(X, Y) = 0$. C) $E(XY) = E(X)E(Y)$.
D) $Var(X + Y) = Var(X) + Var(Y)$. **Answer: A**

(b) (8 point) Let a and b be some constant. Which of the following statement is **generally false**?
A) $Cov(a + bX, a + bY) = b^2E[XY] - b^2E[X]E[Y]$.
B) $Var\,(aX - bY) = a^2Var\,(X) + b^2Var\,(Y) - 2abCov(X, Y)$.
C) $Cov(W, Z) = Corr(X, Y)$, where $W = (X - E(X))/\sqrt{Var(X)}$ and $Z = (Y - E(Y))/\sqrt{Var(Y)}$.
D) Cov$(X, W) = Var(X)$, where $W = (X - E(X))/\sqrt{Var(X)}$. **Answer: D**

6. (10 points) Let Z_1 and Z_2 are two Bernoulli random variables with the probability of success p, where Z_1 and Z_2 are independent, and $Z_i = 0$ with probability $1 - p$ and $Z_i = 1$ with probability p for $i = 1, 2$. Define a random variable $X = Z_1 - Z_2$. Find the mean and the variance of X.

Answer: Note that $E[Z_1] = E[Z_2] = p$ **and** $\mathbf{Var}(Z_1) = \mathbf{Var}(Z_2) = p(1-p)$**. Furthermore, because** Z_1 **and** Z_2 **are independent,** $\mathbf{Cov}(Z_1, Z_2) = 0$**. Therefore,** $E[X] = E[Z_1 - Z_2] = 0$ **and** $\mathbf{Var}(X) = \mathbf{Var}(Z_1 - Z_2) = \mathbf{Var}(Z_1) + \mathbf{Var}(Z_2) + 2\mathbf{Cov}(Z_1, Z_2) = 2p(1-p)$**.**

7. Let X and Y be two discrete random variables. The set of possible values for X is $\{x_1, \ldots, x_n\}$; and the set of possible values for Y is $\{y_1, \ldots, y_m\}$. The joint function of X and Y is given by $p_{ij}^{X,Y} = P(X = x_i, Y = y_j)$ for $i = 1, \ldots n; j = 1, \ldots, m$. The marginal probability function of X is $p_i^X = P(X = x_i) = \sum_{j=1}^{m} p_{ij}^{X,Y}$ for $i = 1, \ldots n$, and the marginal probability function of Y is $p_j^Y = P(Y = y_j) = \sum_{i=1}^{n} p_{ij}^{X,Y}$ for $j = 1, \ldots m$.

   (a) (10 points) For general n and m, prove that $\text{Cov}(X, Y) = 0$ if X and Y are independent. **Please use the summation operator in the proof**.

   Answer: See "Notes on Mathematical Expectation, Variance, and Covariance."

   (b) (10 points) Define $W = (X - E(X))/\sqrt{Var(X)}$. For general n and m, prove that $\text{Corr}(X, W) = 1$.

   Answer: Note that $E(W) = E[(X - E(X))/\sqrt{Var(X)}] = (1/\sqrt{Var(X)})E[X - E(X)] = (1/\sqrt{Var(X)}) \times 0 = 0$**. Then, using** $E(W) = 0$**, we have** $Cov(X, W) = E[(X - E(X))(W - E(W))] = E[(X - E(X))W] = E[(X - E(X))(X - E(X))/\sqrt{Var(X)}] = (1/\sqrt{Var(X)})E[(X - E(X))^2] = (1/\sqrt{Var(X)})Var(X) = \sqrt{Var(X)}$ **and** $Var(W) = E[(W - E(W))^2] = E[W^2] = E[(X - E(X))^2/Var(X)) = (1/Var(X))E[(X - E(X))^2] = (1/Var(X))Var(X) = 1$**. Therefore, it follows from** $Cov(X, W) = \sqrt{Var(X)}$ **and** $Var(W) = 1$ **that**

$$Corr(X, W) = \frac{Cov(X, W)}{\sqrt{Var(X)Var(W)}} = \frac{\sqrt{Var(X)}}{\sqrt{Var(X) \times 1}} = 1.$$