Economics 325–003 **Term I 2016**

Introduction to Empirical Economics **Hiro Kasahara**

Midterm Exam

1. (10 points) In one year, the average stock price of Google Inc. was \$800 with the standard deviation equal to \$100. Using the empirical rule, it can be estimated that approximately 95 % of the stock price of Google Inc. will be in what interval?

2. Let $X = 1$ if a person has completed 4 year university; $X = 0$ otherwise. Let Y be the annual income. For simplicity, assume that Y takes the following three values: 30, 60, and 100 in thousand \$. Suppose that the joint probability of education attainment and annual income of person is given by Table 1.

   (a) (10 points) Find the value of $E[X]$.

   (b) (10 points) Find the value of $E[Y|X = 1] - E[Y|X = 0]$.

Table I: Joint Distribution of University Degree and Annual Incomes (in thousand \$)

	Y = 30	Y = 60	Y = 100
X = 0	0.24	0.12	0.04
X = 1	0.12	0.36	0.12

3. The survey asks eligible voters in the U.S. whether he or she would vote for Clinton or Trump. An individual i's voting preference is recorded as $X_i = 1$ if she/he would vote for Clinton and as $X_i = 0$ if she/he would vote for Trump. Suppose that we randomly select two persons and let their voting preference to be represented by X_1 and X_2, where the probability distribution of X_1 is identical to that of X_2 and is given by

$$X_i = \begin{cases} 0 & \text{with probability } 1-p \\ 1 & \text{with probability } p, \end{cases}$$

for $i = 1, 2$. Here, p represents the population fraction of voters who would vote for Clinton. Define a random variable $\bar{X}$ by $\bar{X} = (X_1 + X_2)/2$.

   (a) (10 points) Derive $E[X_1]$ and $Var[X_1]$.

   (b) (10 points) Derive $E[\bar{X}]$.

   (c) (10 points) Derive $Var[\bar{X}]$.

4. (10 points) For any random variable X, define $W = \frac{X - E(X)}{\sqrt{Var(X)}}$. Derive $E[W]$ and $Var[W]$.

5. (10 points) Let X and Y be two discrete random variables. The set of possible values for X is $\{x_1, \ldots, x_n\}$; and the set of possible values for Y is $\{y_1, \ldots, y_m\}$. The joint function of X and Y is given by $p_{ij}^{X,Y} = P(X = x_i, Y = y_j) \quad \text{for } \ i = 1, \ldots n; j = 1, \ldots, m$. The marginal probability function of X is $p_i^X = P(X = x_i) = \sum_{j=1}^{m} p_{ij}^{X,Y}$ for $i = 1, \ldots n$, and the marginal probability function of Y is $p_j^Y = P(Y = y_j) = \sum_{i=1}^{n} p_{ij}^{X,Y}$ for $j = 1, \ldots m$. Let $g_1(x)$ and $g_2(y)$ are some functions of x and y, respectively. Prove that, if random variable X and Y are independent, then $\text{Cov}(g_1(X), g_2(Y)) = 0$. **Please use the summation operator in the proof for this question**.

6. (10 points) Prove that $Cov(X, Y) = E[XY] - E[X]E[Y]$. In this proof, you don't necessarily use the summation operator.

7. (10 points) Let a, b, and c be some constant. Prove that $Var(aX + bY + c) = a^2 Var(X) + b^2 Var(Y) + 2abCov(X, Y)$. In this proof, you don't necessarily use the summation operator.