**Economics 325–003** **Term I 2017**

**Introduction to Empirical Economics** **Hiro Kasahara**

# Midterm Exam[1]

1. (8 points) In one year, the average stock price of Microsoft Corp. was \$77 with the standard deviation equal to \$5. Using the empirical rule, it can be estimated that approximately 95 % of the stock price of Microsoft Corp. will be in what interval?

2. (8 points) Multiple choice. **Write your answer (i.e., A, B, C, or D) to your booklet.** No explanation necessary. For any two events $A$ and $B$, consider the following two statements:

   1. $(A \cap B) \cap (A \cap \bar{B})$ is empty.
   2. $A = (A \cap B) \cup (A \cap \bar{B})$.

   Which of the followings is true?

   A). Both 1 and 2 are true, B). Both 1 and 2 are false,

   C). 1 is true and 2 is false, D). 1 is false and 2 is true.

3. (10 points) Suppose a personnel officer has 6 candidates to fill 2 positions. Among 6 candidates, 3 candidates are men and 3 candidates are women. If every candidate is equally likely to be chosen, what is the probability that no women will be hired?

4. Suppose you took a blood test for cancer diagnosis and your blood test was positive. This blood test is positive with probability 90 percent if you indeed have a cancer. This blood test is positive with probability 5 percent if you dont have a cancer. In population, it is known that 1 percent of people have cancer. Define $A = \{\text{positive blood test}\}$ and $B = \{\text{having a cancer}\}$.

   (a) (6 points) What is the probability of your having a cancer given that your blood test is positive?

   (b) (6 points) Complete the following table for the joint distribution of $A$ and $B$ and **write your answer to your booklet**, where $\bar{A} = \{\text{negative blood test}\}$ and $\bar{B} = \{\text{not having a cancer}\}$.

Table I: Joint distribution of $A$ and $B$

| | $\bar{B}$ | $B$ | Marginal Prob. |
|---|---|---|---|
| $\bar{A}$ | | | |
| $A$ | | | |
| Marginal Prob | 0.99 | 0.01 | 1.00 |

5. In the household survey, we asked if the household head has completed 4 year university degree or not and asked their annual incomes. Define the random variables $X$ and $Y$ as follows: X = 1 if the household head has completed 4 year university; X = 0 otherwise. Y is the annual incomes. For simplicity, assume that $Y$ takes two values: 50 and 100 in thousand \$. The joint distribution of $X$ and $Y$ is given in Table II.

Table II: Joint Distribution of University Degree and Annual Incomes (in thousand \$)

| | Y = 50 | Y = 100 | Marginal Prob. of X |
|---|---|---|---|
| X = 0 | 0.30 | 0.10 | 0.40 |
| X = 1 | 0.20 | 0.40 | 0.60 |
| Marginal Prob. of Y | 0.50 | 0.50 | 1.00 |

(a) (6 points) Are $X$ and $Y$ stochastically independent? **Prove your claim.** [Full 6 points are given to the proof.]

(b) (6 points) What is the conditional probability mass function of $Y$ given $X = 1$?

(c) (6 points) Find the value of $E[Y|X = 1] - E[Y|X = 0]$.

(d) (6 points) Show that the law of iterated expectations $E_X[E_Y[Y|X]] = E_Y[Y]$ holds for this example.

6. (8 points) The daily stock prices of a company named "Data Science" are known to be normally distributed with a mean of \$100 and a standard deviation of \$10. What proportion of daily stock prices is larger than \$120?

7. (10 points) Suppose that $\{X_1, X_2, ..., X_n\}$ is a random sample (so that $X_1$, $X_2$, ..., $X_n$ are stochastically independent), where $X_i$ takes a value of zero or one with probability $1 - p$ and $p$, respectively. Define $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. Show that $E(\bar{X}) = p$ and $\text{Var}(\bar{X}) = \frac{p(1-p)}{n}$.

8. Let $X$ and $Y$ be two discrete random variables. The set of possible values for $X$ is $\{x_1, \ldots, x_n\}$; and the set of possible values for $Y$ is $\{y_1, \ldots, y_m\}$. The joint function of $X$ and $Y$ is given by $p_{ij}^{X,Y} = P(X = x_i, Y = y_j)$ for $i = 1, \ldots n; j = 1, \ldots, m$. The marginal probability function of $X$ is $p_i^X = P(X = x_i) = \sum_{j=1}^{m} p_{ij}^{X,Y}$ for $i = 1, \ldots n$, and the marginal probability function of $Y$ is $p_j^Y = P(Y = y_j) = \sum_{i=1}^{n} p_{ij}^{X,Y}$ for $j = 1, \ldots m$.

   (a) (10 points) Prove that, if $X$ and $Y$ are stochastically independent, then $Cov(g(X), a + bY) = 0$ for any function $g$ and any constant $a$ and $b$. **Please use the summation operator in the proof for this question**.

   (b) (10 points) Define $Z = \frac{X - E(X)}{\sqrt{Var(X)}}$ and $W = \frac{X - a}{\sqrt{Var(X)}}$ for some constant $a$. Prove that $\text{Corr}(Z, W) = 1$ for any $a$. (You don't necessarily need to use the summation operator).