## Econ 325 Section 001

Assignment 1

## The due date is Tuesday, September 17, before the class starts.

1. Following is a random sample of three $(x, y)$ pairs of data points:

$$
\left(x_{1}, y_{1}\right)=(11,52), \quad\left(x_{2}, y_{2}\right)=(13,72), \quad\left(x_{3}, y_{3}\right)=(15,62)
$$

i.e.,

|  | $i=1$ | $i=2$ | $i=3$ |
| :--- | :---: | :---: | :---: |
| $x_{i}$ | 11 | 13 | 15 |
| $y_{i}$ | 52 | 72 | 62 |

(a) Compute the sample variance of $x$, i.e., $s_{x}^{2}=(1 /(n-1)) \sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}$.
(b) Compute the sample covariance between $x$ and $y$, i.e., $s_{x y}=(1 /(n-1)) \sum_{i=1}^{n}\left(x_{i}-\right.$ $\bar{x})\left(y_{i}-\bar{y}\right)$.
(c) Compute the sample correlation coefficient between $x$ and $y$.
2. In one year, the average stock price of Apple Inc. was $\$ 650$ with the standard deviation equal to $\$ 100$. Using the empirical rule, it can be estimated that approximately $95 \%$ of the stock price of Apple Inc. will be in what interval?
3. Exercise on weighted mean. The final grade point in Econ 325 is computed as a weighted average of assignment $\left(x_{1}\right)$, midterm $\left(x_{2}\right)$, and final exam $\left(x_{3}\right)$. The weights are $w_{1}=0.1, w_{2}=0.3$, and $w_{3}=0.6$ for assignment, midterm, and final exam, respectively. Using the summation sign, we may express the final grade point in terms of $x_{i}$ and $w_{i}$ for $i=1,2,3$ as $\bar{x}=\sum_{i=1}^{n} w_{i} x_{i}=w_{1} x_{1}+w_{2} x_{2}+w_{3} x_{3}$ for $n=3$. Table 2 reports the scores of assignment, midterm, and final exam for two students (A and B). What would be the final grade point for A and B?

Table 2: Grade Points

|  | Assignment $\left(x_{1}\right)$ | Midterm $\left(x_{2}\right)$ | Final Exam $\left(x_{3}\right)$ |
| :--- | :---: | :---: | :---: |
| Student A | 90 | 90 | 46 |
| Student B | 80 | 64 | 90 |

4. Please read "Notes on Summation Operator" posted on the course website. Compute the sum $\sum_{i=1}^{n} x_{i}$, the average $\bar{x}=(1 / n) \sum_{i=1}^{n} x_{i}$, and the sample variance $(1 /(n-$ 1)) $\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}$ when $n=5$ and $x_{i}=i$, i.e., $x_{1}=1, x_{2}=2, \ldots, x_{5}=5$ as follows.
(a) Compute $\sum_{i=1}^{n} x_{i}$
(b) Compute $\bar{x}=(1 / n) \sum_{i=1}^{n} x_{i}$
(c) Compute $s^{2}=(1 /(n-1)) \sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}$
(d) Show that $\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)=0$ for this example.
(e) Show that $\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)=0$ holds for any sequence of numbers $\left\{x_{i}\right\}_{i=1}^{n}$ given that $\bar{x}=(1 / n) \sum_{i=1}^{n=1} x_{i}$.
5. Given two sequences of numbers $\left\{x_{i}\right\}_{i=1}^{n}$ and $\left\{y_{i}\right\}_{i=1}^{n}$ and constant $a$ and $b$, show that the following are true:
(a) $\sum_{i=1}^{n} a x_{i}=a \sum_{i=1}^{n} x_{i}$.
(b) $\sum_{i=1}^{n}\left(x_{i}+y_{i}\right)=\sum_{i=1}^{n} x_{i}+\sum_{i=1}^{n} y_{i}$.
(c) $\sum_{i=1}^{n}\left(a x_{i}+b y_{i}\right)=a \sum_{i=1}^{n} x_{i}+b \sum_{i=1}^{n} y_{i}$.
(d) $\sum_{i=1}^{n} \sum_{j=1}^{n} a b x_{i} y_{j}=a b \sum_{i=1}^{n} x_{i}\left(\sum_{j=1}^{n} y_{j}\right)$
(e) $\sum_{i=1}^{n} \sum_{j=1}^{n} a b x_{i} y_{j}=a b \sum_{j=1}^{n} y_{j}\left(\sum_{i=1}^{n} x_{i}\right)$
