Econ 325 Section 001
Assignment 7

The due date is Tuesday, November 19 before the class starts.

1. State whether each of the following is true or false. No explanation necessary.

   (a) The confidence interval for population parameter θ with confidence level $1-\alpha$ is always given by $\hat{\theta} \pm z_{\alpha/2}\sqrt{Var(\hat{\theta})}$, where $\hat{\theta}$ is an unbiased point estimator of θ and $z_{\alpha/2}$ is the critical value such that $P(Z > z_\alpha) = \alpha$ given $Z \sim N(0,1)$.

   (b) The central limit theorem cannot be applied to discrete random variables.

2. A random sample of 100 men contained 63 in favour of a state constitutional amendment to retard the rate of growth of property taxes. An independent random sample of 100 women contained 55 in favour of this amendment. Suppose that the confidence interval was calculated as $[0.012, 0.148]$ for the difference between the population proportions. What are the significance level and the confidence level of this interval?

3. You are interested in examining the salaries earned by graduate business school students at the end of the first year after graduation. In particular, you are interested in seeing whether there is a difference between men and women graduates' salaries. From a random sample of $n_x = 900$ women, you find the mean salary to be $\bar{X} = \$47,520$ with a standard deviation of $s_x = \$5,633$. From a sample of $n_y = 400$ men, you find the mean salary to be $\bar{Y} = \$44,304$ with a standard deviation of $s_y = \$4,793$. Assume that the sample size is large enough to use the Central Limit Theorem and that s_x and s_y are consistent estimators of population standard deviations of women and men' salaries, where the population variances are assumed to be different between women and men.

   (a) What is a point estimator of the difference between the population mean salary for women and men?

   (b) Compute the variance of $\bar{X} - \bar{Y}$ in terms of the population variances of women and men' salaries.

   (c) Develop a 95% confidence interval for the difference between the population mean salary for women and men, where the sample standard deviations in place of the population standard deviations given that the sample size is large enough. What is the margin of error?

   (d) Based on the confidence interval in the previous question, is there evidence that the population means are different between women and men?

4. The survey asks randomly sampled eligible voters in the U.S. whether he or she would vote for Trump. An individual i's voting preference is recorded as $X_i = 1$ if she/he would vote for Trump and as $X_i = 0$ otherwise. We also define $Y_i = 1$ if she/he would not vote for Trump and $Y_i = 0$ otherwise. Note that $Y_i = 1 - X_i$.

Let p_x and p_y represent a fraction of Trump supporters and Trump non-supporters in population so that $p_x = E[X_i]$ and $p_y = E[Y_i]$. Note that $p_x + p_y = 1$. We are interested in estimating the population difference $p_x - p_y$.

Suppose that we randomly sample $n = 400$ voters and we construct two data sets, $\{X_1, X_2, ..., X_n\}$ and $\{Y_1, Y_2, ..., Y_n\}$, based on the same sample of $n = 400$ voters. Let $\hat{p}_x = \frac{1}{n}\sum_{i=1}^{n} X_i$ and $\hat{p}_y = \frac{1}{n}\sum_{i=1}^{n} Y_i$.

(a) Derive the variance of the difference between $\hat{p}_x$ and $\hat{p}_y$ in terms of p_x and p_y and n.

(b) Suppose that $\hat{p}_x = 0.52$ and $\hat{p}_y = 0.48$. Construct the 95 percent confidence interval for $p_x - p_y$, assuming that n is large enough to apply the Central Limit Theorem.