Econ 326
Assignment 1

The due date for this assignment is Monday, January 18, before the class starts.

Make sure that you read Appendix A and B.

1. Let X and Y be two discrete random variables. The set of possible values for X is $\{x_1, \ldots, x_n\}$; and the set of possible values for Y is $\{y_1, \ldots, y_m\}$. The joint PMF is given by

$$p_{ij}^{X,Y} = P\left(X = x_i, Y = y_j\right), \qquad i = 1, \ldots n; j = 1, \ldots, m.$$

The marginal PMF of X is

$$p_i^X = P\left(X = x_i\right) = \sum_{j=1}^{m} p_{ij}^{X,Y}, \qquad i = 1, \ldots n,$$

and the marginal PMF of Y is

$$p_j^Y = P\left(Y = y_j\right) = \sum_{i=1}^{n} p_{ij}^{X,Y}, \qquad i = 1, \ldots n,$$

Prove the following results:

   (a) If c is a constant, then $Cov\left(X, c\right) = 0$.

   (b) $Cov\left(X, X\right) = Var\left(X\right)$.

   (c) $Cov\left(X, Y\right) = Cov\left(Y, X\right)$.

   (d) $Cov\left(a_1 + b_1 X, a_2 + b_2 Y\right) = b_1 b_2 Cov\left(X, Y\right)$, where a_1, a_2, b_1, and b_2 are some constants.

   (e) If X and Y are independent then $Cov\left(X, Y\right) = 0$.

   (f) $Var\left(X + Y\right) = Var\left(X\right) + Var\left(Y\right) + 2Cov\left(X, Y\right)$.

   (g) $Var\left(X - Y\right) = Var\left(X\right) + Var\left(Y\right) - 2Cov\left(X, Y\right)$.

2. Let $\{x_i : i = 1, \ldots, n\}$ and $\{y_i : i = 1, \ldots, n\}$ be two sequences. Define the averages

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i,$$
$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i.$$

   (a) Show that $\sum_{i=1}^{n}\left(x_i - \bar{x}\right) = 0$.

   (b) Using the result in part (a), show that

$$\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2 = \sum_{i=1}^{n} x_i\left(x_i - \bar{x}\right),$$
$$\sum_{i=1}^{n}\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right) = \sum_{i=1}^{n} y_i\left(x_i - \bar{x}\right) = \sum_{i=1}^{n} x_i\left(y_i - \bar{y}\right).$$

3. Wooldridge, Appendix B, Problems B.2, B.9 and B.10 on pages 745-746.