Economics 628 **Term 1 2017/2018**

Topics in Applied Econometrics I **Hiro Kasahara**

Homework 6

(Due: Wednesday, November 29 at the *start* of the class)

Note: Study groups *discussing* the problems are strongly encouraged. But please write your own answers and submit your own programs (no copy and past from your classmate's program!).

1 Structural Dynamic Discrete Choice Model

Consider a dynamic model of import decision:

$$V(\omega_{t-1}, d_{t-1}) = \max_{d_t \in \{0,1\}} W(\omega_{t-1}, d_{t-1}, d_t) + \xi_t(d_t), \quad (1)$$

$$W(\omega_{t-1}, d_{t-1}, d_t) = E[\tilde{\pi}_\theta(\omega_t, d_t) - FC_\theta(d_t, d_{t-1}) + \beta V(\omega_t, d_t) | \omega_{t-1}, d_{t-1}]. \quad (2)$$

where ω_t is productivity, $(\xi_t(0), \xi_t(1))'$ is import cost shock, d_t is discrete import decision, $\pi_t(\cdot)$ is the profit after maximizing out the variable inputs,

$$FC_\theta(d_t, d_{t-1}) = (\delta_1 + \delta_2(1 - d_{t-1}))d_t$$

is the fixed cost of importing materials where δ_1 is per-period fixed cost while δ_2 is one-time sunk cost of importing.

The stochastic process of ω_t and $(\xi_t(0), \xi_t(1))$ are given by

$$\begin{aligned} \omega_t &= \rho\omega_{t-1} + \epsilon_t, \\ \epsilon_t &\sim_{iid} N(0, \sigma_\epsilon^2), \\ (\xi(0), \xi(1)) &\sim_{iid} \text{Type 1 Extreme value distribution.} \end{aligned}$$

We specify $\tilde{\pi}_\theta(\omega_t, d_t) = \exp(\tilde{\alpha}_0 + \tilde{\alpha}_1\omega_t + \alpha_2 d_t)$ and

$$E[\tilde{\pi}_\theta(\omega_t, d_t)|\omega_{t-1}, d_{t-1}] = E[\exp(\tilde{\alpha}_0+\tilde{\alpha}_1\omega_t+\alpha_2 d_t)|\omega_{t-1}, d_{t-1}] = \exp(\alpha_0+\alpha_1\omega_{t-1}+\alpha_2 d_t) \equiv \pi_\theta(\omega_{t-1}, d_t).$$

with $\alpha_0 = \tilde{\alpha}_0 + 0.5\sigma_\epsilon^2$ and $\alpha_1 = \tilde{\alpha}_1\rho$, where the last equality follows from $\omega_t = \rho\omega_{t-1} + \epsilon_t$ and $E[\exp(\epsilon_t)] = \exp(0.5\sigma_\epsilon^2)$.

Using the property of extreme value distributed variable, (2) is written as

$$V(\omega_{t-1}, d_{t-1}) = \text{Euler's constant} + \ln\left(\sum_{d\in\{0,1\}} \exp(W(\omega_{t-1}, d_{t-1}, d_t))\right) \tag{3}$$

$$W(\omega_{t-1}, d_{t-1}, d_t) = \pi_\theta(\omega_{t-1}, d_t) - FC_\theta(d_t, d_{t-1}) + \beta\int V(\rho\omega_{t-1} + \epsilon', d_t)(1/\sigma_\epsilon)\phi(\epsilon'/\sigma_\epsilon)d\epsilon. \tag{4}$$

Once the fixed point of (7) is solved, then we can compute the conditional choice probabilities by the logit formula as:

$$\Pr_\theta(d_t = 1|\omega_{t-1}, d_{t-1}) = \frac{\exp(\pi_\theta(\omega_{t-1}, 1) - FC_\theta(1, d_{t-1}) + \beta E_{\epsilon'}[V(\rho\omega_{t-1} + \epsilon', 1)])}{\sum_{d'\in\{0,1\}} \exp(\pi_\theta(\omega_{t-1}, d') - FC_\theta(d', d_{t-1}) + \beta E_{\epsilon'}[V(\rho\omega_{t-1} + \epsilon', d')])}. \tag{5}$$

Read "HW6_data" into Matlab, which contains $N \times (T+1)$ panel data for firm's discrete import decisions ("*d_m*") and productivity shock ("*omega*"), where there are $N = 2000$ firms across $T + 1 = 7$ years. Estimate the AR(1) process for ω_{it} and let $\hat{\rho}$ and $\hat{\sigma}_\epsilon^2$ be the estimated parameters [You should get something close to $\hat{\rho} = 0.6$ and $\hat{\sigma}_\epsilon^2 = 0.07$]. In this exercise, you are asked to estimate the parameters $\theta = (\alpha_0, \alpha_1, \alpha_2, \delta_1, \delta_2)'$ while fixing $\hat{\rho}$ and $\hat{\sigma}_\epsilon^2$ at the estimates of the AR(1) process by maximizing the log likelihood function:

$$\mathcal{L}(\theta|\{d_{it}, \omega_{it}\}_{i,t}) = \sum_{i=1}^{N}\sum_{t=1}^{T} \ln(\Pr_\theta(d_{it}|\omega_{i,t-1}, d_{i,t-1})), \tag{6}$$

where $\Pr_\theta(d_{it} = 1|\omega_{i,t-1}, d_{i,t-1})$ is given by (8).

1. Solve the fixed point of (7) by approximate the state space of ω by a set of $N_\omega = 121$ discrete points: $\mathcal{W} = \{-1.5, -1.475, -1.45, ..., -0.025, 0, 0.025, ..., 1.45, 1.475, 1.5\}$ (i.e., set $W = [-1.5 : 0.025 : 1.5]$). Let $\omega^1 = -1.5$, $\omega^2 = -1.475$, ..., $\omega^{120} = 1.475$, and $\omega^{N_\omega} = 1.5$. Let $q^j = (\omega^j + \omega^{j+1})/2$ for $j = 1, ..., N_\omega - 1$ so that q^j is the middle point between ω^j and ω^{j+1}. The AR(1) process of ω is then approximated on the grid as:

$$\Pr(\omega_t = \omega^j|\omega_{t-1} = \omega^i) = \begin{cases} \Phi((q^1 - \rho\omega^i)/\sigma_\epsilon), & \text{if } j = 1 \\ \Phi((q^j - \rho\omega^i)/\sigma_\epsilon) - \Phi((q^{j-1} - \rho\omega^i)/\sigma_\epsilon), & \text{if } 1 < j < N_\omega \\ 1 - \Phi((q^{N_\omega} - \rho\omega^i)/\sigma_\epsilon), & \text{if } j = N_\omega. \end{cases}$$

Compute $\Pr(\omega_t = \omega^j | \omega_{t-1} = \omega^i)$ for $i, j = 1, ..., N_\omega$ and construct a $N_\omega \times N_\omega$ transition matrix, called M_ω, of which (i, j)-th element is $\Pr(\omega_t = \omega^j | \omega_{t-1} = \omega^i)$. We set $\beta = 0.9$.

2. Once the state space is discretized, both the value function V and the conditional choice probabilities $\Pr_\theta(d_t = 1 | \omega_{t-1}, d_{t-1})$ can be expressed by $N_\omega \times 2$ matrices. Write a function m-file which takes θ, M_ω, and $\{\omega^1, ..., \omega^{N_\omega}\}$ as inputs and then produces the fixed point V and the conditional choice probability $\Pr_\theta(d_t = 1 | \omega^j, d_{t-1})$ for $j = 1, ..., N_\omega$.

3. "Discretize" the data on productivity shock, $\{\omega_{it}\}$, by setting ω_{it} equal to the closest grid point among $\{\omega^1, ..., \omega^{N_\omega}\}$, i.e.,

$$\omega_{it}^* = \begin{cases} \omega^1, & \text{if } \omega_{it} \leq q^1 \\ \omega^j, & \text{if } q^j < \omega_{it} \leq q^{j+1} \text{ for some } 1 < j < N_\omega \\ \omega^{N_\omega}, & \text{if } q^{N_\omega} < \omega_{it}. \end{cases}$$

4. Write a function m-file which takes θ, M_ω, and $\{\omega^1, ..., \omega^{N_\omega}\}$, $\{\omega_{it}^*\}$, and $\{d_{it}\}$ as inputs and produces the negative value of log-likelihood function (6) as output. Estimate θ by the maximum likelihood. Report the estimates and their standard errors. You can also use bootstrap.

5. The conditional choice probabilities $\Pr_\theta(d_t = 1 | \omega_{t-1}^*, d_{t-1})$ and the transition matrix of ω_t^* jointly characterize the stochastic process of (d_t, ω_t^*). Using the estimate $\hat{\theta}$ and the transition matrix M_ω, compute the stationary joint distribution of (d_t, ω_t^*) as well as the conditional distribution of d_t given ω_t^* in the long-run.

6. Compute the empirical transition matrix of d_t and report it in the 2×2 table, where the $(1, 1)$-th element is a fraction of firms with $d_t = 0$ given $d_{t-1} = 0$. Compute the predicted transition matrix of d_t under the estimated parameters and compare it with the empirical transition probability. Does the estimated model successfully replicate the dynamic patterns of import status?

7. Suppose that a government unexpectedly and permanently introduces import subsidies in the form of one-time lump-sum transfer for the "first-time" importers at $t = 0$ and, as a result,

the value of δ_2 decreases by 50 %. Analyze how a fraction of importers change over time after the introduction of import subsidies starting from the stationary distribution of (ω_t^*, d_t) by plotting a fraction of importers as y-axis and time as x-axis.[1]

8. Now, suppose that a firm makes an import decision *after* observing ω_t so that the Bellman equation is given by:

$$\begin{aligned} V(\omega_t, d_{t-1}) &= \max_{d_t \in \{0,1\}} W(\omega_t, d_{t-1}, d_t) + \xi_t(d_t), \\ W(\omega_t, d_{t-1}, d_t) &= \tilde{\pi}_\theta(\omega_t, d_t) - FC_\theta(d_t, d_{t-1}) + \beta E[V(\omega_{t+1}, d_t)|\omega_t, d_{t-1}]. \end{aligned}$$

so that

$$\begin{aligned} V(\omega_t, d_{t-1}) &= \text{Euler's constant} + \ln\left(\sum_{d \in \{0,1\}} W(\omega_t, d_{t-1}, d_t)\right) \\ W(\omega_t, d_{t-1}, d_t) &= \tilde{\pi}_\theta(\omega_t, d_t) - FC_\theta(d_t, d_{t-1}) + \beta \int V(\rho\omega_t + \epsilon', d_t)(1/\sigma_\epsilon)\phi(\epsilon'/\sigma_\epsilon)d\epsilon. \end{aligned}$$

The conditional choice probabilities are given by the logit formula as:

$$\Pr_\theta(d_t = 1|\omega_t, d_{t-1}) = \frac{\exp(\tilde{\pi}_\theta(\omega_t, 1) - FC_\theta(1, d_{t-1}) + \beta E_{\epsilon'}[V(\rho\omega_t + \epsilon', 1)])}{\sum_{d' \in \{0,1\}} \exp(\tilde{\pi}_\theta(\omega_t, d') - FC_\theta(d', d_{t-1}) + \beta E_{\epsilon'}[V(\rho\omega_t + \epsilon', d')])}.$$

Estimate a parameter vector $\theta = (\tilde{\alpha}_0, \tilde{\alpha}_1, \alpha_2, \delta_0, \delta_1)'$ and report the standard errors. [Note that the program for solving the Bellman equation is the same as before. When you evaluate the likelihood, you will replace ω_{t-1} with ω_t.]

2 Reduced Form vs. Structural Model

Consider the following model of demand and supply:

$$D_t = \alpha_1 p_t + \alpha_2 y_t + u_{Dt} \tag{7}$$

$$S_t = \beta_1 p_t + \beta_2 w_t + u_{St}. \tag{8}$$

[1] You can do this either by generating the "counterfactual" data set with the estimated model as in homework 2 or use the estimated transition probability of (d_t, ω_t).

We assume that (u_{Dt}, u_{St}) is independent of (y_t, w_t), and independently distributed over time. In equlibrium, we have $q_t = D_t = S_t$. We observe the random sample of $\{q_t, p_t, y_t, w_t\}_{t=1}^T$ generated from this equilibrium model. For your reference, we may define

$$\Gamma = \begin{pmatrix} 1 & -\alpha_1 \\ 1 & -\beta_1 \end{pmatrix} \quad \text{and} \quad \Gamma^{-1} = \begin{pmatrix} \frac{-\beta_1}{\alpha_1-\beta_1} & \frac{\alpha_1}{\alpha_1-\beta_1} \\ \frac{-1}{\alpha_1-\beta_1} & \frac{1}{\alpha_1-\beta_1} \end{pmatrix}.$$

1. Suppose that we consider the following reduced-form specification:

$$q_t = \pi_{11} y_t + \pi_{12} w_t + \tilde{u}_{qt}. \tag{9}$$

   Express π_{11}, π_{12}, and $\tilde{u}_{qt}$ in terms of the structural parameter $\theta = (\alpha_1, \alpha_2, \beta_1, \beta_2)$ and the iid demand and supply shocks, u_{Dt} and u_{St}.

2. Consider the effect of counterfactual policy of consumption tax so that the consumer faces $(1+\tau)p_t$ instead of p_t in their demand equation while the supply equation does not change. The tax revenue is thrown away by the government. Let q_t^{CF} be the counterfactual equilibrium quantity, where we assume that $(y_t, w_t, u_{Dt}, u_{St})$ in this counterfactual remains the same as the actual value. Let $q_t^{CF} - q_t$ be the effect of introducing consumer tax.

   (a) Briefly discuss if the reduced-form parameter in (9) is enough to analyze the effect of the counterfactual consumption tax on equilibrium quantity without knowing the relationship between the reduced-form parameter (π_{11}, π_{12}) and the structural parameter θ.

   (b) Derive $q_t^{CF} - q_t$ in terms of the structural parameter θ and the iid demand and supply shocks, u_{Dt} and u_{St}.

   (c) Discuss how to compute the standard error for $E[q_t^{CF} - q_t]$.