Econ 325 Section 003: Worksheet for Estimation[1]

Name and Student No._____

Question 1 *In the U.S. presidential election, let p be the population fraction of voters who support Trump. Consider a random sample of* 10000 *voters* $\{X_1, X_2, ..., X_{10000}\}$*, where* $X_i = 1$ *if the i-th individual is Trump supporter and* $X_i = 0$ *if s/he is Clinton supporter. Consider the following two estimators:*

$$\hat{p}_1 = \frac{1}{10}\sum_{i=1}^{10} X_i \quad \textit{and} \quad \hat{p}_2 = \frac{1}{10000}\sum_{i=1}^{10000} X_i.$$

$\hat{p}_1$ only uses the first 10 observations in the sample. $\hat{p}_2$ uses all of 10000 *observations. Compute the variance of $\hat{p}_1$ and $\hat{p}_2$. Is it true that $\hat{p}_2$ is more efficient than $\hat{p}_1$?*

Answer

Question 2 *Given a random sample of $n = 2$ with $Var(X_i) = \sigma^2$. consider an estimator of μ:*

$$\hat{\mu} = \beta X_1 + (1 - \beta)X_2.$$

Compute the variance of $\hat{\mu}$ in terms of β and σ^2. What is the value of β that gives the smallest variance of $\hat{\mu}$?

Answer

Question 3 *A sample of 25 light bulb from a large normal population has a mean life length of 1500. We know that the population standard deviation is 10. Determine a 95% confidence interval for the true mean length of life in the population.*

Answer

Question 4 *A sample of 25 light bulb from a large normal population has a mean life length of 1500. We don't know the population standard deviation but the sample standard deviation is computed as 10. Determine a 95% confidence interval for the true mean length of life in the population.*

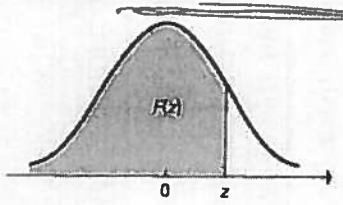
Answer

Question 5 *The survey was conducted between Oct. 20 and 24, 2016, in Florida after the third and final presidential debate. The survey result shows that, among 1166 likely registered voters who support either Clinton or Trump, there are 602 Clinton voters and 564 Trump voters. What is the 95 precent confidence interval for the population fraction of Clinton voters?*

Answer

APPENDIX TABLES

Table 1 Cumulative Distribution Function of the Standard Normal Distribution

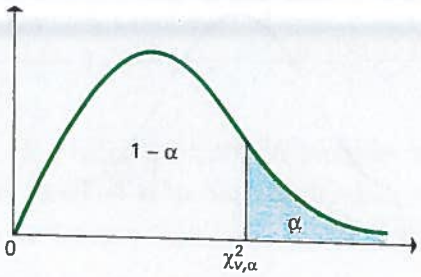


z	F(z)	z	F(z)	z	F(z)	z	F(z)	z	F(z)	z	F(z)
.00	.5000										
.01	.5040	.31	.6217	.61	.7291	.91	.8186	1.21	.8869	1.51	.9345
.02	.5080	.32	.6255	.62	.7324	.92	.8212	1.22	.8888	1.52	.9357
.03	.5120	.33	.6293	.63	.7357	.93	.8238	1.23	.8907	1.53	.9370
.04	.5160	.34	.6331	.64	.7389	.94	.8264	1.24	.8925	1.54	.9382
.05	.5199	.35	.6368	.65	.7422	.95	.8289	1.25	.8944	1.55	.9394
.06	.5239	.36	.6406	.66	.7454	.96	.8315	1.26	.8962	1.56	.9406
.07	.5279	.37	.6443	.67	.7486	.97	.8340	1.27	.8980	1.57	.9418
.08	.5319	.38	.6480	.68	.7517	.98	.8365	1.28	.8997	1.58	.9429
.09	.5359	.39	.6517	.69	.7549	.99	.8389	1.29	.9015	1.59	.9441
.10	.5398	.40	.6554	.70	.7580	1.00	.8413	1.30	.9032	1.60	.9452
.11	.5438	.41	.6591	.71	.7611	1.01	.8438	1.31	.9049	1.61	.9463
.12	.5478	.42	.6628	.72	.7642	1.02	.8461	1.32	.9066	1.62	.9474
.13	.5517	.43	.6664	.73	.7673	1.03	.8485	1.33	.9082	1.63	.9484
.14	.5557	.44	.6700	.74	.7704	1.04	.8508	1.34	.9099	1.64	.9495
.15	.5596	.45	.6736	.75	.7734	1.05	.8531	1.35	.9115	1.65	.9505
.16	.5636	.46	.6772	.76	.7764	1.06	.8554	1.36	.9131	1.66	.9515
.17	.5675	.47	.6803	.77	.7794	1.07	.8577	1.37	.9147	1.67	.9525
.18	.5714	.48	.6844	.78	.7823	1.08	.8599	1.38	.9162	1.68	.9535
.19	.5753	.49	.6879	.79	.7852	1.09	.8621	1.39	.9177	1.69	.9545
.20	.5793	.50	.6915	.80	.7881	1.10	.8643	1.40	.9192	1.70	.9554
.21	.5832	.51	.6950	.81	.7910	1.11	.8665	1.41	.9207	1.71	.9564
.22	.5871	.52	.6985	.82	.7939	1.12	.8686	1.42	.9222	1.72	.9573
.23	.5910	.53	.7019	.83	.7967	1.13	.8708	1.43	.9236	1.73	.9582
.24	.5948	.54	.7054	.84	.7995	1.14	.8729	1.44	.9251	1.74	.9591
.25	.5987	.55	.7088	.85	.8023	1.15	.8749	1.45	.9265	1.75	.9599
.26	.6026	.56	.7123	.86	.8051	1.16	.8770	1.46	.9279	1.76	.9608
.27	.6064	.57	.7157	.87	.8078	1.17	.8790	1.47	.9292	1.77	.9616
.28	.6103	.58	.7190	.88	.8106	1.18	.8810	1.48	.9306	1.78	.9625
.29	.6141	.59	.7224	.89	.8133	1.19	.8830	1.49	.9319	1.79	.9633
.30	.6179	.60	.7257	.90	.8159	1.20	.8849	1.50	.9332	1.80	.9641

Table 1 Cumulative Distribution Function of the Standard Normal Distribution Continue

z	F(z)	z	F(z)	z	F(z)	z	F(z)	z	F(z)
1.81	.9649	2.21	.9864	2.61	.9955	3.01	.9987	3.41	.9997
1.82	.9656	2.22	.9868	2.62	.9956	3.02	.9987	3.42	.9997
1.83	.9664	2.23	.9871	2.63	.9957	3.03	.9988	3.43	.9997
1.84	.9671	2.24	.9875	2.64	.9959	3.04	.9988	3.44	.9997
1.85	.9678	2.25	.9878	2.65	.9960	3.05	.9989	3.45	.9997
1.86	.9686	2.26	.9881	2.66	.9961	3.06	.9989	3.46	.9997
1.87	.9693	2.27	.9884	2.67	.9962	3.07	.9989	3.47	.9997
1.88	.9699	2.28	.9887	2.68	.9963	3.08	.9990	3.48	.9997
1.89	.9706	2.29	.9890	2.69	.9964	3.09	.9990	3.49	.9998
1.90	.9713	2.30	.9893	2.70	.9965	3.10	.9990	3.50	.9998
1.91	.9719	2.31	.9896	2.71	.9966	3.11	.9991	3.51	.9998
1.92	.9726	2.32	.9898	2.72	.9967	3.12	.9991	3.52	.9998
1.93	.9732	2.33	.9901	2.73	.9968	3.13	.9991	3.53	.9998
1.94	.9738	2.34	.9904	2.74	.9969	3.14	.9992	3.54	.9998
1.95	.9744	2.35	.9906	2.75	.9970	3.15	.9992	3.55	.9998
1.96	.9750	2.36	.9909	2.76	.9971	3.16	.9992	3.56	.9998
1.97	.9756	2.37	.9911	2.77	.9972	3.17	.9992	3.57	.9998
1.98	.9761	2.38	.9913	2.78	.9973	3.18	.9993	3.58	.9998
1.99	.9767	2.39	.9916	2.79	.9974	3.19	.9993	3.59	.9998
2.00	.9772	2.40	.9918	2.80	.9974	3.20	.9993	3.60	.9998
2.01	.9778	2.41	.9920	2.81	.9975	3.21	.9993	3.61	.9998
2.02	.9783	2.42	.9922	2.82	.9976	3.22	.9994	3.62	.9999
2.03	.9788	2.43	.9925	2.83	.9977	3.23	.9994	3.63	.9999
2.04	.9793	2.44	.9927	2.84	.9977	3.24	.9994	3.64	.9999
2.05	.9798	2.45	.9929	2.85	.9978	3.25	.9994	3.65	.9999
2.06	.9803	2.46	.9931	2.86	.9979	3.26	.9994	3.66	.9999
2.07	.9808	2.47	.9932	2.87	.9979	3.27	.9995	3.67	.9999
2.08	.9812	2.48	.9934	2.88	.9980	3.28	.9995	3.68	.9999
2.09	.9817	2.49	.9936	2.89	.9981	3.29	.9995	3.69	.9999
2.10	.9821	2.50	.9938	2.90	.9981	3.30	.9995	3.70	.9999
2.11	.9826	2.51	.9940	2.91	.9982	3.31	.9995	3.71	.9999
2.12	.9830	2.52	.9941	2.92	.9982	3.32	.9996	3.72	.9999
2.13	.9834	2.53	.9943	2.93	.9983	3.33	.9996	3.73	.9999
2.14	.9838	2.54	.9945	2.94	.9984	3.34	.9996	3.74	.9999
2.15	.9842	2.55	.9946	2.95	.9984	3.35	.9996	3.75	.9999
2.16	.9846	2.56	.9948	2.96	.9985	3.36	.9996	3.76	.9999
2.17	.9850	2.57	.9949	2.97	.9985	3.37	.9996	3.77	.9999
2.18	.9854	2.58	.9951	2.98	.9986	3.38	.9996	3.78	.9999
2.19	.9857	2.59	.9952	2.99	.9986	3.39	.9997	3.79	.9999
2.20	.9861	2.60	.9953	3.00	.9986	3.40	.9997	3.80	.9999

Table 7 Cutoff Points of the Chi-Square Distribution Function



For selected probabilities α, the table shows the values $\chi^2_{\nu,\alpha}$ such that $P(\chi^2_\nu > \chi^2_{\nu,\alpha}) = \alpha$, where χ^2_ν is a chi-square random variable with ν degrees of freedom. For example, the probability is .100 that a chi-square random variable with 10 degrees of freedom is greater than 15.99.

ν	α									
	.995	.990	.975	.950	.900	.100	.050	.025	.010	.005
1	0.0^4393	0.0^3157	0.0^3982	0.0^2393	0.0158	2.71	3.84	5.02	6.63	7.88
2	0.0100	0.0201	0.0506	0.103	0.211	4.61	5.99	7.38	9.21	10.60
3	0.072	0.115	0.216	0.352	0.584	6.25	7.81	9.35	11.34	12.84
4	0.207	0.297	0.484	0.711	1.064	7.78	9.49	11.14	13.28	14.86
5	0.412	0.554	0.831	1.145	1.61	9.24	11.07	12.83	15.09	16.75
6	0.676	0.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

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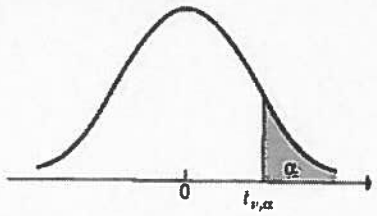
Table 8 Upper Critical Values of Student's t Distribution with ν Degrees of Freedom



For selected probabilities, α, the table shows the values $t_{\nu,\alpha}$ such that $P(t_\nu > t_{\nu,\alpha}) = \alpha$, where t_ν is a Student's t random variable with ν degress of freedom. For example, the probability is .10 that a Student's t random variable with 10 degrees of freedom exceeds 1.372.

	Probability of Exceeding the Critical Value					
ν	0.10	0.05	0.025	0.01	0.005	0.001
1	3.078	6.314	12.706	31.821	63.657	318.313
2	1.886	2.920	4.303	6.965	9.925	22.327
3	1.638	2.353	3.182	4.541	5.841	10.215
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.893
6	1.440	1.943	2.447	3.143	3.707	5.208
7	1.415	1.895	2.365	2.998	3.499	4.782
8	1.397	1.860	2.306	2.896	3.355	4.499
9	1.383	1.833	2.262	2.821	3.250	4.296
10	1.372	1.812	2.228	2.764	3.169	4.143
11	1.363	1.796	2.201	2.718	3.106	4.024
12	1.356	1.782	2.179	2.681	3.055	3.929
13	1.350	1.771	2.160	2.650	3.012	3.852
14	1.345	1.761	2.145	2.624	2.977	3.787
15	1.341	1.753	2.131	2.602	2.947	3.733
16	1.337	1.746	2.120	2.583	2.921	3.686
17	1.333	1.740	2.110	2.567	2.898	3.646
18	1.330	1.734	2.101	2.552	2.878	3.610
19	1.328	1.729	2.093	2.539	2.861	3.579
20	1.325	1.725	2.086	2.528	2.845	3.552
21	1.323	1.721	2.080	2.518	2.831	3.527
22	1.321	1.717	2.074	2.508	2.819	3.505
23	1.319	1.714	2.069	2.500	2.807	3.485
24	1.318	1.711	2.064	2.492	2.797	3.467
25	1.316	1.708	2.060	2.485	2.787	3.450
26	1.315	1.706	2.056	2.479	2.779	3.435
27	1.314	1.703	2.052	2.473	2.771	3.421
28	1.313	1.701	2.048	2.467	2.763	3.408
29	1.311	1.699	2.045	2.462	2.756	3.396
30	1.310	1.697	2.042	2.457	2.750	3.385
40	1.303	1.684	2.021	2.423	2.704	3.307
60	1.296	1.671	2.000	2.390	2.660	3.232
100	1.290	1.660	1.984	2.364	2.626	3.174
∞	1.282	1.645	1.960	2.326	2.576	3.090

NIST/SEMATECH e-Handbook of Statistical Methods, http://www.itl.nist.gov/div898/handbook/, September 2011.