An Introduction to Logic and Proofs*

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Created: August 7, 2004
This Version: August 19, 2004

1 Logic

Mathematical economics, as the title suggests is a course on the use of mathematics in economics. The thinking within the framework of mathematics falls under the class of reasoning known as deductive reasoning. That is, conclusions are drawn using logic based on statements assumed true. Therefore, given true assumptions, the conclusions *need* to be proven true. As a result, if the conclusions are in direct contradiction with reality, the error is not in the theory but rather, on the assumptions.[1] Since economic models often take on assumptions for simplicity and clarity in exposition, the usage of deductive logic within economics is vastly important.

1.1 Propositions

A *proposition* is a statement which is either true or false. For example, 10 is a natural number, $2 + 4 = 6$, and today is August 7th, 2004 are all propositions. However, not all sentences are propositions. What is your

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[1]For example, one may assume the following two statements; (i) Birds cannot fly, (ii) A crow is a bird. Then, it is logically correct to conclude that crows cannot fly. This conclusion is true given the assumptions are, which, are obviously in direct contradiction to everyday observation.

name? and $y^2 = 9$ are examples of sentences which are not propositions.[2] Another type of sentence which is not a proposition is that of a *paradox* such as "This sentence is false."[3]

The three examples of propositions provided above are known as *simple* or *atomic* propositions. *Compound* propositions are more complex with the usage of connectives. In other words, atomic propositions are joined, using connectives, to make compound propositions. For example, "it is sunny today and today is August 7th, 2004" is a compound proposition using the two atomic propositions "it is sunny today" and "today is August 7th, 2004." Naturally, "and" is the connective used to make the compound sentence valid.

Atomic propositions are often by a letter, say, P. Hence, if P is a proposition, it has a truth value assigned to it. Then a compound sentence may be created using a conjunction, disjunction or negation. For example, suppose P and R are atomic propositions. Then the conjunction of P and R is $P \wedge R$ read "P and R." This compound proposition is true when exactly both P and R are true. The disjunction of P and R is $P \vee R$ read "P or R." The disjunction is true when at least P or R is true. The negation of a proposition R is written $\sim R$, $-R$ or $\neg R$ read "it is not the case that R," or simply as "not R" and is true exactly when R is false.

Example 1 *Consider the following two propositions:*

$$P = \text{"The sky is blue."}$$
$$R = \text{"I bought two apples today."}$$

The the statement "The sky is blue and I bought two apples today" may be denoted as $P \wedge R$. The statement "The sky is blue or I bought two apples today" is denoted $P \vee R$ and lastly, "It is not the case that I bought two apples today" is denoted $\sim R$.

The examples of compound propositions made use of only one connective. However, it is often the case that more complicated propositions are formed

[2] A *truth value* (i.e., true of false) cannot be assigned to questions hence it is not a proposition. For the statement $y^2 = 9$, on the other hand, a truth value cannot be assigned unless the value of y is known.

[3] Note that if the above sentence is true then it implies that it is false. Similarly, if it is false, then it implies it is true. A contradiction in itself.

using many connectives. Thus, it will be useful to introduce the notion of a *proposition form.* A propositional form is a statement using finitely many logical symbols and letters. A valid propositional form is known as a *well-formed formula.* It is then natural to conclude that not all propositional forms are well-formed formula. For example, suppose P, R, and S are atomic propositions. Then $\sim P \sim R$ is not a well-formed formula while $P \wedge R \wedge S$ is.

Therefore, given a well-formed formula, or a valid propositional form, it is often useful to understand when it is true. For this purpose, one may refer to truth tables which assigns a truth value according to the connectives used.[4]

Example 2 *Suppose P and R are atomic propositions. Then the truth table for the connectives $\wedge$ and $\vee$ are:*[5]

P	R	$P \wedge R$
T	T	T
T	F	F
F	T	F
F	F	F

P	R	$P \vee R$
T	T	T
T	F	T
F	T	T
F	F	F

At this conjuncture, it is also useful to take note of the notion *equivalent.* Two propositional forms are said to be equivalent *if and only if* they have the same truth tables.[6] It is then easy to see that two atomic propositions, P and R, are, by definition, equivalent even if they have no relationship.

Example 3 *Suppose we have the following two atomic propositions:*

$$P = \text{“Today is August 9th, 2004.”}$$
$$R = \text{“}7 + 6 = 13.\text{”}$$

Then P and R are equivalent.

[4] The use of truth tables is sometimes referred to as *Model Theory.*

[5] For clarity, T is used to denote true and F is used to denote false.

[6] The notion of an “if and only if” statement has yet been covered. For a more detailed discussion, see Section 1.2.

Aside from the trivial example on equivalence among propositions above, the skill to express propositions which are deemed logically equivalent is very useful when dealing with proofs.[7] This is due to the fact that it may be much easier to prove the propositional form $P \vee (R \wedge P)$ rather than P, which are equivalent compound propositions.[8]

One should also take note of the notion of *tautologies* and its negation a *contradiction*. A propositional form is termed a tautology if it takes on a true value regardless of the propositions which constitute the propositional form. As an illustrative example, the propositional form $P \vee \sim P$ is a tautology since it is always true. One can confirm the intuition by replacing P with any atomic propositions such as "Today is August 9th, 2004." It is clearly always true since today is either August 9th, 2004 or it is not. One or the other must be true!

Given the definition of a tautology, a contradiction is therefore a propositional form which is always false. Since a contradiction is defined as the negation of a tautology, it is easy to see that $\sim (P \vee \sim P)$ is a contradiction. Again, using the example above, it is obviously false if one makes the claim "It is not the case that today is either August 9th, 2004 or it is not."

1.2 Conditional and Biconditional Sentences

Given our discussion of propositional forms and the usage of connectives in Section 1.1, one may be tempted to think that there may exist other connectives in our language. For example, there are propositional forms comprised of atomic propositions which may not use the conjunction, disjunction, or negation connectives. This, in fact is true as we have yet discussed the most used connectives within economics and mathematics, in general. These are the *conditional* and *biconditional* sentences.

Given two atomic propositions P and R, $P \Rightarrow R$ read "P implies R" or "if P, then R" is called a conditional sentence where P is referred to as the antecedent and R the consequent. The propositional form $P \Rightarrow R$ is true exactly when the antecedent is false or the consequent is true. In summary, the truth table is illustrated below.

[7] See, for example Section 2.

[8] To illustrate this is simple. Interested readers should attempt Exercise 3.

P	R	$P \Rightarrow R$
T	T	T
T	F	F
F	T	T
F	F	T

It is interesting to observe that the conditional sentence $P \Rightarrow R$ can be true even if the antecedent is false. This follows since a false antecedent automatically makes the logical relationship between P and R irrelevant. For example, it will be useful to convince yourself that any relationship between the two atomic propositions $P =$"Humans can fly" and $R =$"My name is John" break down when we consider the conditional sentence $P \Rightarrow R$.[9]

There exists a few propositional forms related to conditional sentences that are also worth mentioning. The first is known as the *converse*. The converse for the conditional sentence $P \Rightarrow R$ is $R \Rightarrow P$. The second is known as the *contrapositive* and is $\sim R \Rightarrow \sim P$ for the proposition $P \Rightarrow R$. Upon scrutiny, it is easy to see that $P \Rightarrow R$ is equivalent to its contrapositive whereas this is not true for its converse.[10]

Example 4 *Consider the following two atomic propositions:*

$$P = \text{“Jack ate an apple.”}$$
$$R = \text{“There are apples left.”}$$

Then $P \Rightarrow \sim R$ is "If Jack ate an apple, then it is not he case that there are apples left." The converse is "If it is not the case that there are apples left, then Jack ate an apple." Similarly, the contrapositive is "If there are apples left, then it is not the case that Jack ate an apple."

While a conditional sentence $P \Rightarrow R$ is not equivalent to its converse, they can take on the same truth values. For example the conditional sentence "If I

[9]The conditional sentence is read "If humans can fly then my name is John." This sentence is true since the antecedent $P =$"Humans can fly" is false. Hence, regardless of whether my name is John, it does not depend on the truthfulness of whether humans can fly. Alternatively, we can analyze the conditional sentence as such; Since humans cannot fly, it does not matter what it implies since that state of the world will never be realized.

[10]See Exercise 6.

am sick, then I take medicine," and its converse "If I take medicine, then I am sick" are both true.[11] Propositions where the conditional sentence $P \Rightarrow R$ and its converse $R \Rightarrow P$ are true are known as *biconditional sentences* and are denoted $P \Leftrightarrow R$ and read "P if and only if R".[12] It is then clear that biconditional sentences are true whenever both P and R have the same truth values. The truth table for biconditional sentences are illustrated below.

P	R	$P \Leftrightarrow R$
T	T	T
T	F	F
F	T	F
F	F	T

Note that conditional and biconditional sentences are often used in economics but may be disguised in equivalent propositional forms. For example, the central bank is conducting open market operations so the interest rate will move is equivalent to saying if the central bank is conducting open market operations, then the interest rate will move.[13] Alternatively, the demand curve is downward sloping is equivalent to the compound proposition that a consumer will purchase more if and only if the price goes down since a demand curve is by definition the mapping of the relationship between price and quantity and we take the law of demand to be true.

It is useful to be acquainted with the following theorem without proof.[14]

Theorem 1 *Suppose P, R and S are atomic propositions. Then the propositional form appearing in column A is logically equivalent to the corresponding*

[11]Of course, the truthfulness of this claim is based on the assumption that one only takes medicine when ill.

[12]Note that biconditional statements are commonly abbreviated as P iff R.

[13]Note that this is an example of a conditional sentence rather than a biconditional sentence since the interest rate moving does not imply that the central bank is conducting open market operations.

[14]The tedious proof of this is left as an exercise to the reader. See Exercise 7.

propositional form in Column B.

A	B
$P \Leftrightarrow R$	$(P \Rightarrow R) \wedge (R \Rightarrow P)$
$\sim (P \vee R)$	$(\sim P) \wedge (\sim R)$
$\sim (P \wedge R)$	$(\sim P) \vee (\sim R)$
$\sim (P \Rightarrow R)$	$P \wedge \sim R$
$\sim (P \wedge R)$	$P \Rightarrow \sim R$
$P \wedge (R \vee S)$	$(P \wedge R) \vee (P \wedge S)$
$P \vee (R \wedge S)$	$(P \vee R) \wedge (P \vee S)$

Familiarity with Theorem 1 is highly encouraged as equivalent representation of typical propositional forms will prove to be fruitful when dealing with proofs later on.[15]

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1.3 Existential and Universal Quantifiers

Recall from our discussion on propositions in Section 1.1 that the sentence $y^2 = 9$ is not a proposition. The reason was due to the fact that no truth value may be assigned to such a sentence unless the value of y is known. For this reason, such sentences are known as *predicate* and sometimes referred to as open sentences. More specifically, a predicate is a sentence which contains variables so that its truthfulness may not be established unless the sentence is evaluated with particular objects.[16]

Since it is generally the case that not all objects will result in a truthful predicate, the collection of objects that will is generally smaller. Hence, it is convenient to call the collection of such objects the *truth set* for the open sentence. However, the truth set is determined from a restricted class of objects known as the *universe*. In other words, the universe determines the admissible choices for the truth set and is generally apparent given the predicate involved. For example, suppose we have the predicate "This page has x number of words." Then it is clear that the universe contains the set of natural numbers $\mathbb{N}$. However, it is not always the case that the universe is apparent. For example, suppose the predicate is "$y < 2$", then if the universe

[15]See Section 2.

[16]For example, the variable in the predicate $y^2 = 9$ is y. Hence, $y = 7$ and $y = 3$ are examples of objects for establishing the truthfulness of the predicate.

is $\mathbb{N}$ then the truth set is $\{1\}$ where as if the universe is $\mathbb{R}$ then the truth set is the open interval $(-\infty, 2)$.

Aside from actually replacing the variable in a predicate with an object for evaluation, in order to make it a proposition, an alternative, and equally important, way is to use quantifiers. The *existential quantifier* denoted by $\exists$ is used to construct the proposition $\exists x P(x)$ read "There exists a x from the universe such that $P(x)$“, where $P(x)$ is the predicate. Similarly, the *universal quantifier* denoted by $\forall$ is used to construct the proposition $\forall x P(x)$ read "For all x in the universe, $P(x)$". Note that $\exists x P(x)$ is true when there is at least one x from the universe such that $P(x)$ is true and $\forall x P(x)$ is true if precisely all x in the universe makes $P(x)$ true.

Example 5 *Suppose the universe is restricted to all human-beings in Canada and we have the predicate $P(x) =$ "x's name is George." Then the proposition $\exists x P(x)$ would read; there exists a human-being in Canada such that his/her name is George. Similarly, the proposition $\forall x P(x)$ would read; for all human-beings in Canada, his/her name is George. It is then empirically/observationally clear that $\exists x P(x)$ is true whereas $\forall x P(x)$ is not.*

Much like before, it is useful to familiarize yourself with the following Theorem.

Theorem 2 *Suppose $P(x)$ is a predicate where x is the variable. Then:*[17]

$$\sim \exists x P(x) \quad \textit{is equivalent to} \quad \forall x \sim P(x)$$

and:

$$\sim \forall x P(x) \quad \textit{is equivalent to} \quad \exists x \sim P(x)$$

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1.4 Exercises

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[17]Note that equivalence of quantified sentences is established if and only if they have the same truth value for all universes.

Exercise 1 *For each of the following sentence, state whether it is an atomic proposition or a compound proposition or neither. (i) Today is a hot day, (ii) 9+8-7=44, (iii)* $y = 6x$ *or* $x = (1/6)y$*, and (iv) Is it too loud or too quiet in here?*

Exercise 2 *Suppose* P*,* R*, and* S *are atomic propositions. Construct the truth tables for the propositional form; (i)* $(P \wedge R) \vee S$*, (ii)* $P \wedge R \wedge S$*, and (iii)* $P \vee R \vee S$*.*

Exercise 3 *Suppose* P *and* R *are atomic propositions. Show using truth tables that the functional form* $P \vee (R \wedge P)$ *is equivalent to* P*.*[18]

Exercise 4 *Suppose* P *and* R *are atomic propositions. Are the following propositional forms a tautology, a contradiction or neither? (i)* $P \wedge \sim (R \vee P)$*, (ii)* $(P \wedge R) \vee (\sim P \wedge \sim R)$*, and (iii)* $P \wedge (\sim R \wedge \sim P)$*.*

Exercise 5 *Suppose* P *and* R *are two atomic propositions. Show that* $P \Rightarrow R$ *is equivalent to* $(\sim P) \vee R$*.*

Exercise 6 *Suppose* P *and* R *are two atomic propositions. Show that* $P \Rightarrow R$ *is equivalent to its contrapositive and that* $P \Rightarrow R$ *is not equivalent to its converse. For the latter, illustrate with an example using a language by choosing* P *and* R *such that* $P \Rightarrow R$ *is true and its converse is not.*[19]

Exercise 7 *Prove Theorem 1.*

Exercise 8 *Assume the universe is the set of real numbers* $\mathbb{R}$*. Are the following propositions true? (i)* $\forall x, x + 2 \geq 0$*, (ii)* $\exists x, \sqrt{x} = -8$*, and (iii)* $\forall x, 2 + 2 = x$*.*

Exercise 9 *Assume the universe is the set of real numbers* $\mathbb{R}$*. Show that the following is true.* $\forall x, \exists y, \sqrt{(x-1)^2 - (y+2)^2} = 0$*.*

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[18] It should be noted that the question has an embedded hint in that it suggests that one shows the equivalence between the two propositional forms using truth tables.

[19] As noted above, while a conditional sentence and its converse are not equivalent, it is not true that they can never take on the same truth values.

2 Proofs

Given our discussion of propositions and the use of connective and quantifiers to make valid propositional forms, the next important step is in the determination of its truthfulness. Note that truth tables for a certain propositional form only indicate to us when a proposition (atomic or compound) is true or false given some presumed truthfulness or falsefulness of the inherent atomic propositions. One way of thinking about a truth table is therefore the generic characterization of truth for a given propositional form. Therefore, if one claims a propositional form is true, it must be proven. Since unproven claims may not be true, the technique of proving things is an important tool to learn since it is a valid logical deduction given the assumptions/axioms or other previously proven results.

The notion of a correct proof is sometimes harder to grasp than one imagines. For example, if one claims that $\forall x, x^2 + 7 + 15$ is true and proves by stating that for example, if $x = 3$, then $3^2 + 7 = 15$ and hence true, it is clear that for $x = 2$, $2^2 + 7 = 11 \neq 15$. This above fallacy is a typical mistake that most people make when constructing proofs in that proofs cannot be complete by example.[20]

Therefore, this section is devoted into broad introduction to the techniques of proofs.

2.1 Techniques of Proofs

We begin by introducing one of the simplest valid arguments known as the *modus ponens*. The form of this proof is quite simple. Suppose P and R are atomic propositions and suppose we want to claim that R is true. Then given $P \Rightarrow R$, it is evident that if P is true, so must be R.[21] This is illustrated

[20] The example in this case will be to consider only the case when $x = 3$ if the claim is that $\forall x, x^2 + 7 = 15$. As another illustrative example, if one claims that "All crows are black." Then one cannot prove this claim simply by showing me a black crow. If one wants to challenge this claim, then a non-black crow (perhaps an albino) must be presented. This claim, actually relies on empirical truth rather than absolute truth. In other words, since one cannot check every single crow in the world, the truthfulness of this claim lies in empirical consistency. This is referred to as *inductive reasoning*. Hence, there may exist a state of the world where there exists a crow which is not black.

[21] Refer to the truth table for the conditional sentence $P \Rightarrow R$.

below.

1.	$P \Rightarrow R$
2.	P

Conclusion	R

Hence for a modus ponens, one essentially has to establish the truthfulness of P and the conditional sentence $P \Rightarrow R$ to have proven the result R is true. This then suggests the following tautology; Suppose P and R are propositions. Then:

$$P \wedge (P \Rightarrow R) \Rightarrow R$$

is a tautology.

The above is an example of a proof technique known as a *direct proof* where the proof is provided directly from the assumed facts, namely, that $P \Rightarrow R$ and P is true. Alternatively, suppose we want to prove that $P \Rightarrow R$ is true. Then a direct proof of this will only need to show that the false element associated with the propositional form $P \Rightarrow R$ cannot happen.[22] This coincides to the case where if P is true and R is false. Hence, the direct proof will need to exhaust this possibility. More specifically, the proof will have the features:

1. Assume P is true.
2. $\vdots$ (Some logically valid argument.)
3. Therefore, R.

Note that the proof deduces from the assumption that P is true that R must also be true. Hence, it is sufficient to conclude that $P \Rightarrow R$.

Example 6 *Suppose a and b are even numbers. Prove that $a \times b$ is also a even number.*

Notice that we can define the above into the following propositions; Let $P =$ "a and b are even numbers." and $R =$ "$a \times b$ is a even number." Then in essence, we are proving that $P \Rightarrow R$.[23]

[22]Note that this differs from the above example as we are now establishing the truthfulness of $P \Rightarrow R$ rather than the truthfulness of R.

[23]A number x is said to be even if for some integer t, $x = 2t$. Furthermore, note that P is actually a compound proposition using the conjunction connective.

Proof: *Suppose P is true. Then by definition of a even number, there exist integers y and z such that $a = 2y$ and $b = 2z$. Hence, $a \times b = 2y \times 2z = 4y \times z = 2 \times (2yz)$. Since y and z are integers, so must be $2yz$. Therefore, R is true.*

While a direct proof for certain propositional forms may be easy at times, they may be hard to construct for others. For example, it may be hard to deduce logically from a proposition P that R is true rendering the proof of $P \Rightarrow R$ incomplete. Therefore, one may try to prove this conditional sentence by contraposition.

Recall from Section 1.2 that the contraposition for the conditional sentence $P \Rightarrow R$ is $\sim R \Rightarrow \sim P$. Also recall that a conditional sentence and its contraposition are equivalent, hence, the truth tables are identical. Therefore, a direct proof of the contraposition for the conditional sentence $P \Rightarrow R$ will have one assuming $\sim R$ is true and then to logically deduce that $\sim P$ is also true. In essence, this then implies that $\sim R \Rightarrow \sim P$ is true and hence $P \Rightarrow R$.

Example 7 *Suppose a and b are integers. Show that if $a \times b$ is odd then a and b are odd.*[24]

Proof: *Suppose is it not the case that a and b are odd. Then either a or b is even or both. Let $a = 2y$ where y is an integer. If b is odd, then let $b = 2x \pm 1$. Then, $a \times b = 2y \times (2x \pm 1) = 4xy - 2y = 2(2xy \pm y)$ and hence it is even. Therefore, suppose b is even and so $b = 2x$. Then $a \times b = 2y \times 2x = 4xy = 2(2xy)$ and hence even. Therefore, we have shown that if it is not the case that a and b are odd, then it is not the case that ab is odd. Hence, we have proven the statement if $a \times b$ is odd, then a and b are odd.*

Yet, another useful method of proof is known as a *proof by contradiction.* Consider the proof regarding the truthfulness of a proposition P. Then, it suffices to show that $\sim P$ implies R and $\sim R$ where R is some other proposition. In other words, since $R \wedge \sim R$ is by definition equivalent to $\sim (R \vee \sim R)$, it is a contradiction. Given the tautology $P \vee \sim P$, if $\sim P$

[24] An integer x is odd if there exists some integer t such that $x = 2t + 1$ or $x = 2t - 1$.

cannot be true (given the contradiction in $R \wedge \sim R$,) P must be true. The steps for the proof of the truthfulness of some proposition P is summarized below.

1. Suppose $\sim P$.
2. $\vdots$
3. Therefore, R.
4. $\vdots$
5. Therefore, $\sim R$.
6. Therefore, $R \wedge \sim R$, a contradiction.
7. Hence, P is true.

Example 8 *Suppose a and b are real numbers. Prove that if $a < b$ then $2a < 2b$.*
Proof: *Suppose $a < b$ and $2a \geq 2b$.*[25] *Since $2 > 0$, then dividing both sides by two for $2a \geq 2b$ implies $a \geq b$. Hence, $(a < b) \wedge (a \geq b)$ is true, a contradiction. Therefore, if $a < b$ then $2a < 2b$.*

The last type of proof that one should familiarize themselves with is the *proof using the principle of mathematical induction.* For statements which are recursive in nature, we may want to prove such statements inductively. A proof using the principle of mathematical induction begins with "Let $S = \{n \in \mathbb{N} :$ the statement is true for $n\}$. Then the following steps are taken; (i) Show that $1 \in S$, (ii) Show that if $n \in S$ then $n + 1 \in S$ (The inductive set S exists), and lastly, (iii) By the principle of mathematical induction, $S = \mathbb{N}$."[26]

This method of proof is extremely useful for propositions like $\forall n \in \mathbb{N}$, show that $n^2 \leq (n + 1)$, or $\forall n \in \mathbb{N}$, $n + 3 < 5n^2$. We illustrate the latter of the two in the following example.

Example 9 *Prove that for all $n \in \mathbb{N}$, $n + 3 < 5n^2$.*
Proof: *Consider $n = 1$. It is clear that $1 + 3 < 5$ and therefore, this claim is satisfied for $n = 1$ (This is step (i) above where we have shown that $1 \in S$).*

[25] Note that this is the negation for the proposition "If $a < b$ then $2a < 2b$."
[26] The notion of sets and membership of items to sets has yet been discussed. Refer to the notes of Set Theory for a more detailed exposition.

Now suppose this is true for some $n \in \mathbb{N}$. Then for $n+1$ we have:

$$(n+1)+3 = n+3+1 < 5n^2+1$$

where the last inequality follows from the assumption that $n+3 < 5n^2$. Furthermore, it follows that:

$$5n^2+1 < 5n^2+10n+5 = 5(n+1)^2$$

which follows from the assumption that $n \in \mathbb{N}$. Therefore, the above implies that $(n+1)+3 < 5(n+1)^2$ and hence, the claim is true for $n+1$ as well (This is step (ii) above where we have shown that if $n \in S$ then $n+1 \in S$). Hence, we have proved by induction that the claim is true for all $n \in \mathbb{N}$ (This is step (iii) above).

In essence, one may think of a prove by induction to be like a domino set. We simply have to knock off the first piece (Step (i)) and show that the rest of domino set is aligned so that all the other pieces fall (Step (ii)).

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2.2 Exercises

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Exercise 10 *Suppose a is an integer. Prove that if a^2 is odd then a is odd.*

Exercise 11 *Suppose a and b are integers. Prove that if a and b are even, then $a+b$ is even.*

Exercise 12 *Prove that there exists integers a and b such that $3a-4b = 73$.*

Exercise 13 *Prove that there exists an odd integer a and a even integer b such that $3a+4b = 17$.*

Exercise 14 *Prove that for all $n \in \mathbb{N}$, $2^n \geq 1+n$.*

Exercise 15 *Is the following true or false? If true, prove. If false, show a counterexample. For all $n \in \mathbb{N}$, $n^2+1 < n^3$.*

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